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MEMBERSHIP

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

THE RESERVOIR AS A FLOOD-CONTROL STRUCTURE

BY GEORGE R. CLEMENS,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

This paper presents in an abbreviated form an analysis of the methods of obtaining flood control by means of reservoirs. It begins with a general definition of the flood-control problem and a classification of all possible methods of control. The detailed analysis deals only with the reservoir solution of the problem. For the purpose of bringing to the mind of the reader the various facts that are necessary to consider, a rough outline of the general procedure followed in approaching a specific flood-control problem is given. This may appear elementary to the experienced engineer, but should serve to catalog the various items that must be considered.

The design of a reservoir system is analyzed by discussing: (*a*) The method of operation to be selected; (*b*) testing one method for a specific reservoir; (*c*) transmission of reservoir reduction to area to be protected with a detailed analysis of methods of determining valley storage effect; and (*d*) typical examples of three other methods of storage operation.

The operation of the reservoir for various other purposes, such as water power, irrigation, navigation, and water supply combined with flood control is next considered. This is followed by a discussion of the place of the reservoir in a system with other structures.

In conclusion, there is a summary of the considerations affecting reservoir control. Although most of the general principles are well known to engineers it is believed the paper serves a useful purpose in correlating and classifying methods and in presenting a number of new angles in the practical problem of determining the possible operation of a reservoir system.

NOTE.—Discussion on this paper will close in September, 1934, *Proceedings*.

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Introduction

When rainfall occurs that provides more run-off than can be carried within the normal channels of existing streams a flood results. The excess water overflows the valley lands, invades developed areas, and causes destruction of property or, in extreme cases, loss of human life. There is no known method of regulating the rainfall itself. Nature alone controls the cycle from sea to sky to earth. Man's efforts are confined to attempts at guiding the water on that part of its course from earth back to the sea. The regulation of the waters that would cause floods presents the flood-control problem.

Many structures are built to control water: Waterways of proper depth and width provide arteries of transportation; water controlled and discharged through water-wheels provides power; water may be caught in storage basins and distributed for irrigation or water supply; and water that is an actual or potential source of damage or danger to property or to human life may be controlled to prevent floods. When this control of water involves improvement or reclamation of property not damaged in its present condition, the control is termed "reclamation," and when it involves the prevention of flood damage this is termed "flood control."

Historical.—The common law holds that flood water is a common enemy and that any individual may protect himself against it. From the beginning of recorded history this principle has been followed and a wide range of structures and methods has been developed for combating the "common enemy." Every country in the world has some flood problems. The Po River of Italy, the Seine of France, the Rhine of Germany, the Thames of England, the Yellow of China, and many others have absorbed the efforts of countless engineers in the struggle for their control. In the United States the most notable struggle has been that with the Mississippi River. As early as 1717 the settlers at New Orleans, La., began to protect themselves against overflow. The work has grown in magnitude until at the end of 1933 the Federal Government had spent $351 372 000 in an effort to control this great river. Many smaller but still important projects in this country have included the Sacramento River of California, the Arkansas River of Colorado, the Colorado River of Arizona and Nevada, and the Miami River of Ohio.

The individual farmer may cut a ditch to divert the flood water of a small creek from his field. He may build a small levee to prevent this flood water from invading his fields or buildings in the flats. However, when the area to be protected becomes of considerable size it has been the usual experience that the works of protection are of such magnitude as to be entirely beyond the means of the individual to finance or to construct. Thus, individuals have grouped themselves together in districts. The rights of the individual have become merged with those of the group, and works have been developed which provide the greatest benefit to the majority of the group. Different groups frequently have conflicting interests and still larger units are formed in an attempt to arrive at a solution which will be of greatest benefit to the greatest number. This grouping has grown to State-wide control in some cases and, in the case of the Mississippi River, to control by the Federal Government.

The most comprehensive flood-control investigations in recent years have been those conducted by the United States War Department under the Corps of Engineers, United States Army. These were largely an outgrowth of the great Mississippi River flood of 1927. For several years prior to that flood a comprehensive study of water resources was being planned by the Engineer Corps. This study was to cover navigation, flood control, water power, and irrigation. The 1927 Mississippi River flood emphasized the importance of these studies as relating to flood control and was largely responsible for the fact that the money for these studies was practically doubled and work undertaken on a much larger and more comprehensive scale than previously anticipated. Many of the studies have been completed and a wealth of information has been collected for practically all the important streams of the United States. General plans have been formulated for most streams and any agency confronted with the development of a flood-control plan should find itself with a large part of the groundwork completed and the necessary basic information available to proceed with the detailed development of any specific project, or the co-ordination of any series of projects.

Possible Methods of Flood Control.—Floods are rather common. In one form or another they are experienced by practically every individual and community. Consequently, the methods of protection against a flood causing damage are many and varied. The following general classification of methods is offered:

(*A*) Works to retain or regulate flood waters above the area where damage is caused: (1) Storage reservoirs, with controlled outlets operated in accordance with a pre-determined plan; and (2) retarding reservoirs, with uncontrolled operation of outlets designed so that their maximum flow will not exceed the channel capacity.

(*B*) Works to provide secondary channels through affected areas: (1) Improvement of secondary existing channels; (2) constructing side channels; (3) overbank diversions; and (4) outlets combining overbank and channel diversions.

(*C*) Works to increase the capacity of existing channels through affected areas: (1) Levee systems; (2) cut-offs; and (3) channel deepening and improvement.

This paper will be confined to an analysis of Group (*A*). Naturally, combinations of Groups (*A*), (*B*), and (*C*) are often feasible and desirable. This analysis will deal with the function of the reservoir—its limitations and advantages—and will discuss its place with regard to the whole, and the useful functions it may be expected to perform.

Preliminary Investigation Required

The engineer is ordinarily called upon to design a flood-protection project after a disastrous flood has occurred. In some few cases potential flood damage is anticipated, and a project is designed and constructed prior to the occurrence of what might have been a major damaging flood. In any case the value of a project is the sum of money that those protected from flood

damage will pay as representing the benefit occurring by reason of insured protection from future damage. To determine this value the area subject to overflow must be blocked out and the existing and probable future concentration of all values in various units of it must be determined. Study should be made of the manner in which floods have caused damage in the past; economic surveys should be made of the amount of such damage; and estimates should be prepared of the possible damage that might be caused by future floods. By considering all these factors an estimate may be reached of the probable value of future flood protection which could be used as a basis for assessing the cost against those benefited.

When a reservoir solution (Group (A)) is to be considered many more data concerning flood volumes must be obtained than for other plans. The reservoirs either store the flood water during the danger period (Group (A–1)), or so retard the flow during this period that it does not exceed the capacity of the existing channels (Group (A–2)). Both studies require the collection of many hydrological and hydraulic data.

Outline of Preliminary Investigations for a Typical Stream

An outline of the data required and collected[2] for a consideration of the flood-control problems of a part of the Ouachita River, in Arkansas and Louisiana, in so far as they pertain to reservoir analysis, will best illustrate the procedure in approaching such a project. Fig. 1 is a map of the Ouachita River Basin. By field investigation it was found that the valley lands from Malvern, Ark., to Camden, Ark., have been damaged by floods. To determine the sum that might be spent economically for a flood-protection project an economic survey was made of past flood damage in this danger zone and an estimate was prepared of the value of the elimination of this damage to the centers of population and the farm lands in the valley.

Boundaries of the area to be protected having been located, the first problem of the investigation was to determine the capacity of the existing flood channel. As no field survey was available, a modified hydrographic survey was made which included: (*a*) The determination of river sections at critical points; (*b*) the determination of valley cross-sections at controlling points; (*c*) the low-water river-slope profile; (*d*) location of high-water marks, and determination of high-water profile; (*e*) elevation of controlling top banks; and (*f*) location and size of tributary streams.

Gauges were established and read at critical points along the main stream and on the principal tributaries. It was learned that old gauge records were available for a considerable period for Hot Springs, Malvern, Arkadelphia, and Camden, Ark. These records were collected, and all gauges were referenced to a common datum plane. Discharge measurements were made at each of these key gauge stations and the relation of stage to discharge was determined. In most cases this was plotted as a single curve, as shown in Fig. 2. At Camden, however, it was necessary to develop curves to correct for the variations of local slope as in Fig. 3. To use this type of curve successfully,

[2] Rept. on Ouachita River and Tributaries, Arkansas and Louisiana, submitted to Congress through official channels.

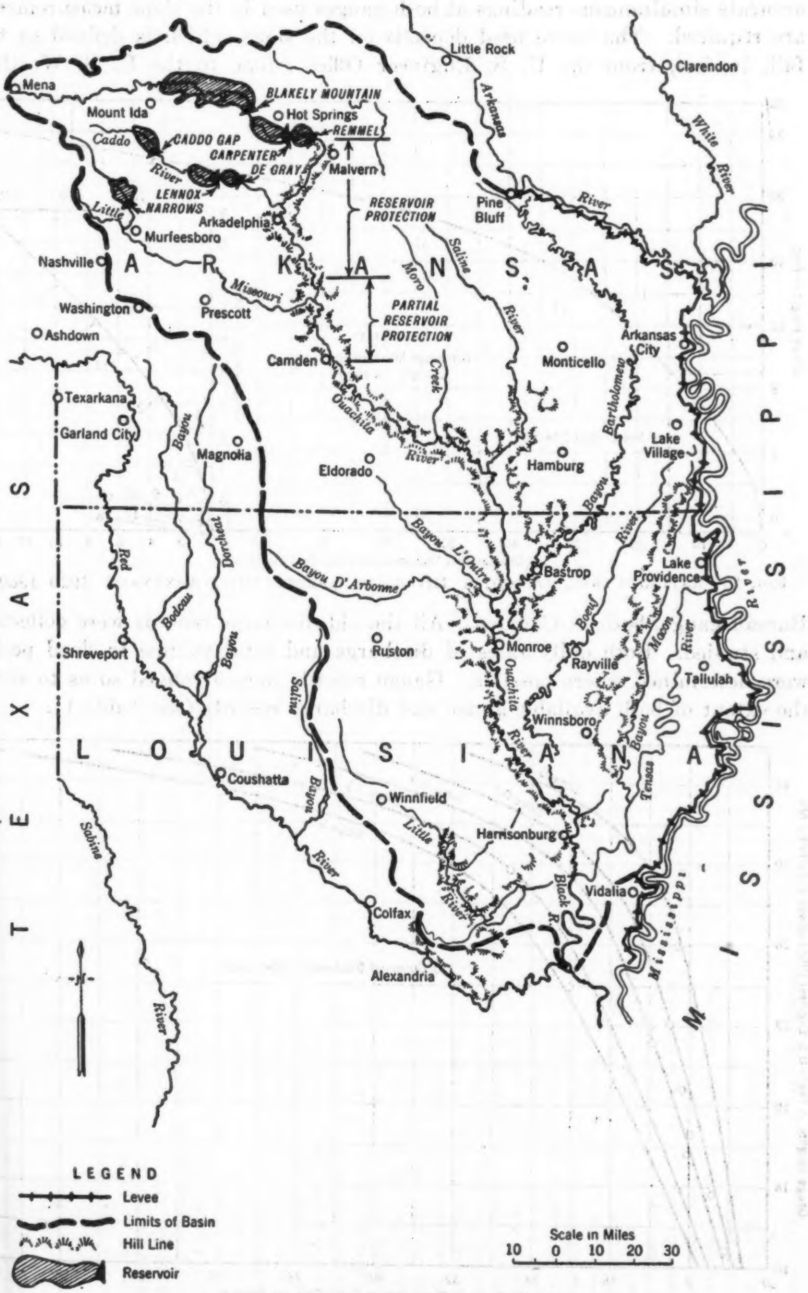


FIG. 1.—OUACHITA RIVER, ARKANSAS AND LOUISIANA

accurate simultaneous readings at both gauges used in the slope measurements are required. The curve used depends on the slope, which is defined as the fall, in feet, from the U. S. Engineer Office gauge to the U. S. Weather

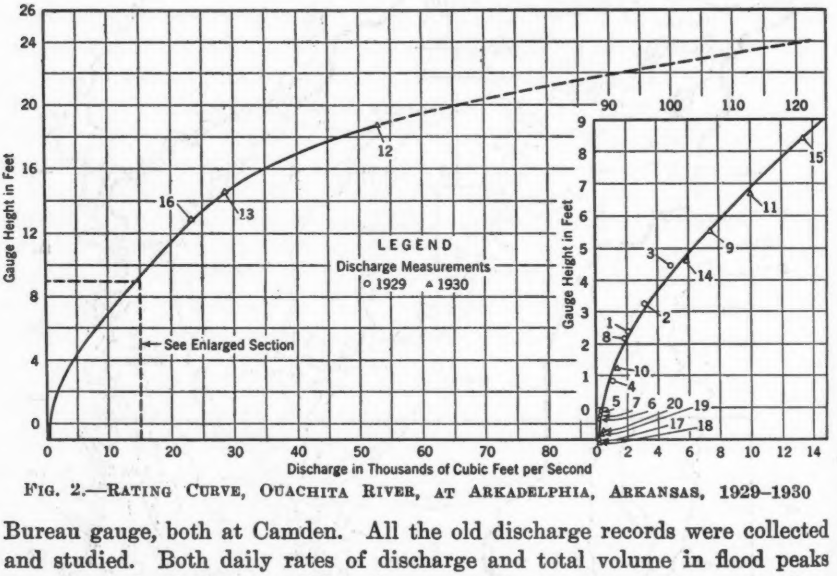


FIG. 2.—RATING CURVE, OUACHITA RIVER, AT ARKADELPHIA, ARKANSAS, 1929–1930

Bureau gauge, both at Camden. All the old discharge records were collected and studied. Both daily rates of discharge and total volume in flood peaks were determined where possible. Gauge records were arranged so as to show the extent of both available gauge and discharge records (see Table 1).

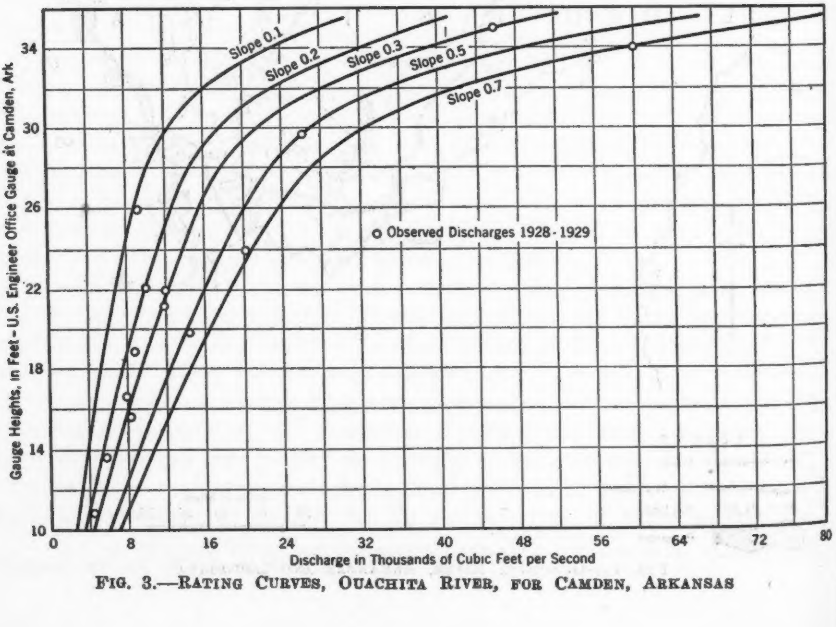
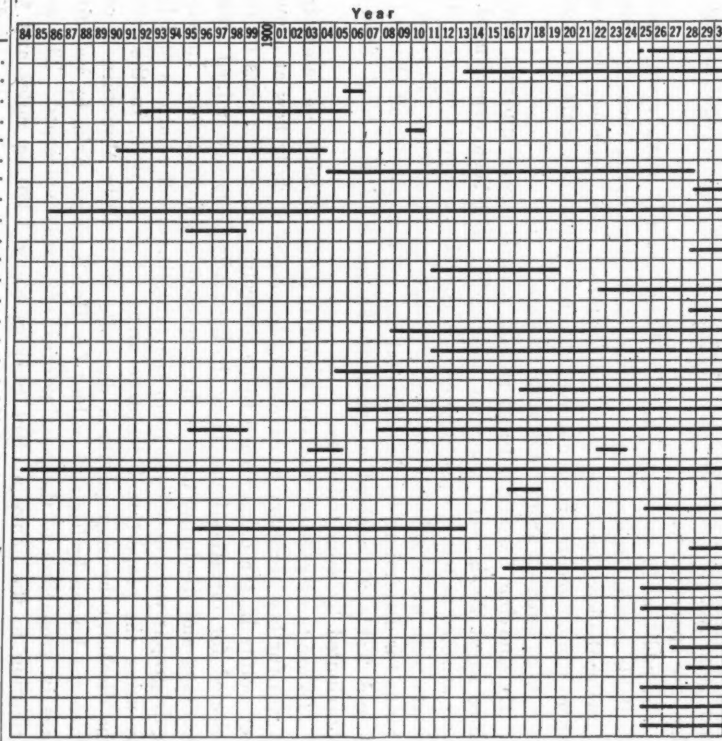


FIG. 3.—RATING CURVES, OUACHITA RIVER, FOR CAMDEN, ARKANSAS

TABLE 1.—GAUGE RECORDS, OUACHITA RIVER AND TRIBUTARIES



Item No.	Gauge (1)	River (2)	Maximum Record Date (3)	Maximum Record Feet (4)	Minimum Record Date (5)	Minimum Record Feet (6)
1	Acme, La.	Black	1927	.....	9-18-25	6.9
2	Arkadelphia, Ark.	Ouachita	4-21-27	23.9	9-1-25	-0.7
3	Arkadelphia, Ark.	Ouachita	1-23-06	19.7	8-13-05	1.3
4	Black River Station, La.	Black	4-20-03	52.3	10-28-97	0.8
5	Blanco Springs, Ark.	Ouachita	4-4-10	1.80	12-6-10	1.0
6	Camden, Ark.	Ouachita	3-23-94	43.25	9-21-04	1.95
7	Camden, Ark.	Ouachita	1-6-07	42.9	8-17-24	1.1
8	Camden, Ark.	Ouachita	5-21-30	41.05	10-5-30	3.2
9	Camden, Ark.	Ouachita	4-25-27	41.0	8-5-29	3.3
10	Champagnolle, Ark.	Ouachita	3-25-97	33.3	9-25-96	0.0
11	Cold Spring Bar, Ark.	Ouachita	5-17-29	23.8	9-4-29	-0.08
12	Fletchers Landing, Ark.	Ouachita	4-16-12	82.9	12-10-12	47.4
13	Hot Springs, Ark.	Ouachita	5-16-23	43.9	9-30-28	4.5
14	Jonesville, La.	Black	6-15-29	52.9	8-28-30	6.5
15	Lock and Dam No. 2	Ouachita	5-18-27	67.2	11-24-22	7.6
16	Lock and Dam No. 3	Ouachita	5-15-27	57.0	11-22-14	1.1
17	Lock and Dam No. 4	Ouachita	5-4-27	48.9	11-21-16	-0.4
18	Lock and Dam No. 5	Ouachita	5-3-27	45.8	8-23-18	0.0
19	Lock and Dam No. 6	Ouachita	4-30-27	43.0	8-5-14	-2.2
20	Lock and Dam No. 8	Ouachita	4-27-27	38.9	12-6-12	0.0
21	Malvern, Ark.	Ouachita	5-9-03	20.2	12-4-04	0.1
22	Monroe, La.	Ouachita	5-4-27	48.7	10-1-97	0.0
23	Newport Landing, Ark.	Ouachita	....	.....	....	.....
24	Remmeldam, Ark.	Ouachita	1927	.....	....	.....
25	Riverton, La.	Ouachita	3-29-03	50.3	9-25-96	0.9
26	Beekman, La.	BayouBartholomew	5-21-30	23.6	10-7-29	1.2
27	Clayton, La.	Tensas	5-17-27	67.3	8-24-25	14.8
28	Delhi, La.	Bayou Macon	5-10-27	34.6	11-25-28	0.0
29	Farmerville, La.	Bayou D'Arbonne	5-19-27	34.5	9-6-25	0.7
30	Farmerville, La.	Bayou D'Arbonne	5-21-30	36.1	7-27-30	10.6
31	Murfreesboro, Ark.	Little Missouri	5-3-30	14.0	7-23-30	1.7
32	Warren, Ark.	Saline	5-19-30	25.9	8-29-29	1.8
33	Warren, Ark.	Saline	4-25-27	26.5	9-23-25	0.7
34	Girard, La.	Boeuf	5-12-30	29.4	6-10-26	0.7
35	Wilmot, Ark.	BayouBartholomew	5-10-27	25.9	9-27-29	1.1

Supplementary to Table 1, the following data are to be noted: Item No. 9, Column (4), a maximum reading of 46.0 ft was estimated on May 12, 1882; Item No. 12, Columns (4) and (6), elevations are referred to mean Gulf level; Item No. 23, Columns (3) to (6), the river overtopped this gauge in March, 1917, and April, 1918; it reached a point below the bottom of the gauge in December, 1916, and June, 1918; Item No. 24, Columns (4) to (6), an automatic gauge is installed at this point, due to the intermittent discharge caused by a power-house; Item No. 26, Column (4), on April 17, 1927, the occurrence of an estimated maximum reading of 26.75 ft is recorded; and, Item No. 29, Columns (3) and (4), the maximum gauge reading and the date are approximate.

In addition to the gauge and discharge records a study was made of all available rainfall records. For convenience in studying these records, a map showing the location of all hydrological stations was prepared. In many cases where run-off records were not available for all points for the greatest storms they were built up from these rainfall data.

By studying the rainfall and run-off records it was apparent that two or three great floods were the most severe that had been experienced. These were selected as test storms to be used in checking reservoir design. Isohyetal charts were quite valuable in studying these great storms. For the greatest storms (as determined by the isohyetal charts) rainfall run-off charts (see Fig. 4 and Table 2) enabled a study to be made of the total flood volume contributed by each tributary.

To use these greatest floods of record as design floods for a reservoir system it was further necessary to develop hydrographs of daily flow at a number of key points in the danger zone to be protected and at the sites of reservoirs selected for investigation. These hydrographs were determined from gauge readings and discharge data or by computations from them. In some cases hydrographs were developed from the rainfall run-off relations and in others by using the proportional drainage areas. For example, in the latter case, the discharge hydrograph for a dam site in Blakely Mountain is computed from the known hydrograph values for Hot Springs, Ark., by reducing readings in proportion to the relative drainage areas concerned. The area contributing to the gauge at Hot Springs being 1 420 sq miles and the area contributing to the gauge at the dam site, 1 190 sq miles, the readings are reduced in the ratio of 1 190 : 1 420, or 0.837.

If a hydrograph is plotted using a logarithmic scale for the discharge ordinates, the proportional drainage-area method becomes much simplified. Multiplication is then accomplished by addition and hydrographs of proportional areas can be determined from known hydrographs by marking off uniform distances above or below the known curve.

A field reconnaissance was made of the entire drainage basin to determine all possible reservoir sites. All available maps of sites that were considered promising were collected and examined. From the general data obtained by this reconnaissance and from existing maps a tentative system (see Fig. 1) was calculated to control the waters to the extent shown to be necessary by the run-off studies.

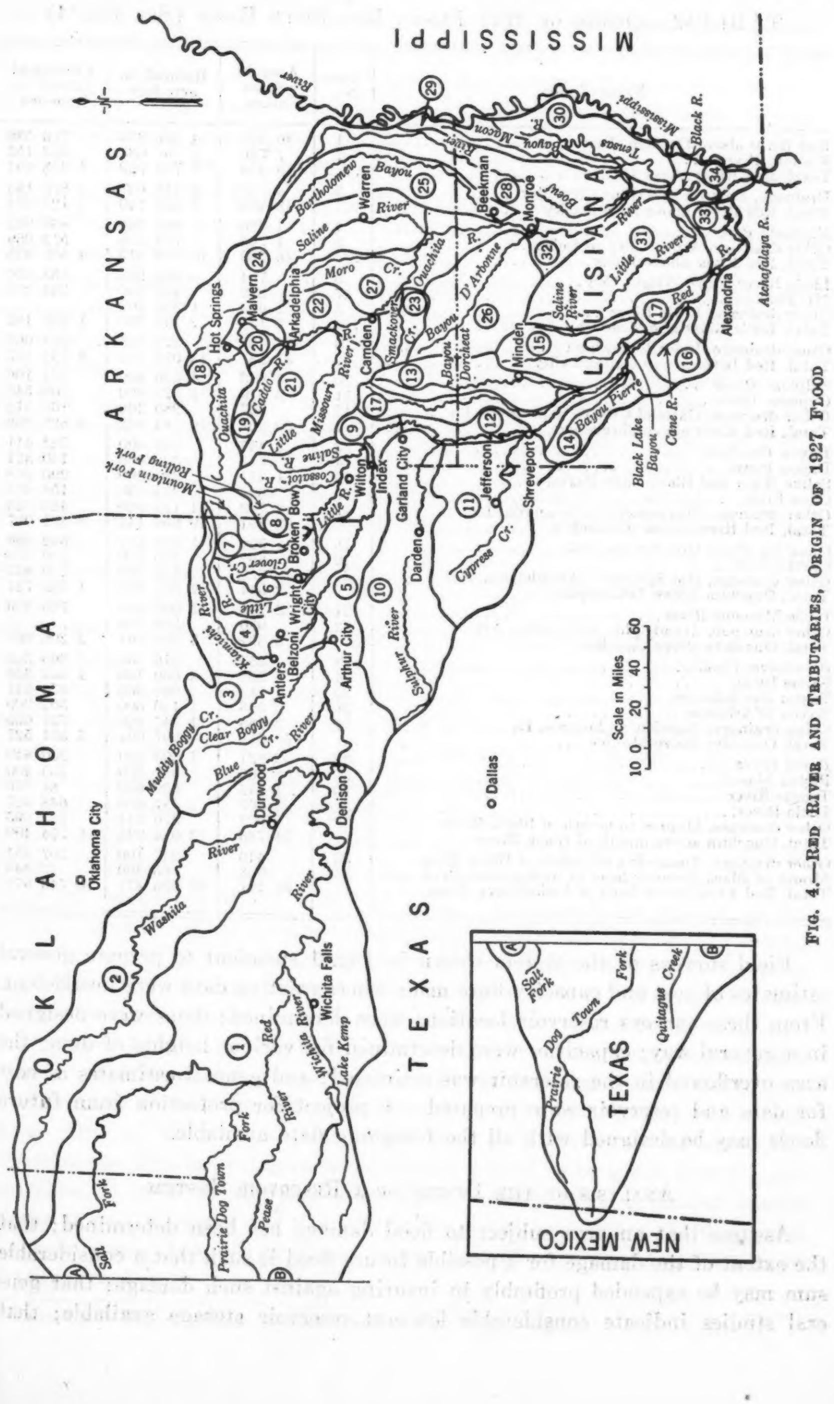


FIG. 4.—RED RIVER AND TRIBUTARIES, ORIGIN OF 1927 FLOOD

TABLE 2.—ORIGIN OF 1927 FLOOD, RED RIVER BASIN (SEE FIG. 4)

Name of area	Area No.	Area, in square miles	Rainfall, in acre-feet	Computed run-off, in acre-feet
Red River above Denison, Tex.	1	30 542	4 306 639	716 539
Washita River	2	7 786	2 397 083	532 152
Total, Red River above Denison	....	38 328	6 703 722	1 248 691
Drainage, Denison to Arthur City, Tex.	3	4 780	2 143 027	1 877 183
Total, Red River above Arthur City	....	43 108	8 846 749	3 125 874
Kiamichi River	4	1 726	887 362	859 942
Other drainage, Arthur City to Index	5	1 746	774 508	572 069
Total, Red River above Index	....	46 580	10 508 619	4 557 885
Little River above Wright City	6	704	436 900	383 030
Mt. Fork	7	812	438 300	384 258
Other drainage, Little River	8	2 618	1 356 100	....
Total, Little River above mouth	......	4 134	2 231 300	1 956 181
Other drainage, Index to Garland	9	423	272 923	218 061
Total, Red River above Garland City	......	51 137	13 012 842	6 732 127
Sulphur River	10	3 983	1 656 400	924 106
Cypress River	11	3 510	1 625 900	709 543
Other drainage, Garland City to Shreveport, La.	12	667	289 300	163 512
Total, Red River above Shreveport	......	59 297	16 584 442	8 529 288
Bayou Dorcheat	13	1 447	603 600	263 411
Bayou Pierre	14	785	322 000	140 521
Saline River and Black Lake Bayou	15	1 471	665 600	290 468
Cane River	16	628	363 600	158 675
Other drainage, Shreveport to Alexandria, La.	17	2 223	1 119 900	488 724
Total, Red River above Alexandria	......	65 851	19 659 142	9 871 087
Ouachita above Hot Springs, Ark.	18	1 304	1 020 591	882 490
Caddo River	19	492	300 000	259 410
Other drainage, Hot Springs to Arkadelphia, Ark.	20	515	365 000	240 827
Total, Ouachita above Arkadelphia	......	2 311	1 685 591	1 382 727
Little Missouri River	21	2 100	1 069 900	705 920
Other drainage, Arkadelphia to Camden, Ark.	22	980	589 200	....
Total, Ouachita above Camden	......	5 391	3 344 691	2 206 903
Smackover Creek	23	557	316 100	208 563
Saline River,	24	3 281	2 050 100	1 352 656
Bayou Bartholomew	25	1 912	988 000	474 141
Bayou D'Arbonne	26	2 348	1 150 800	502 209
Other drainage, Camden to Monroe, La.	27	2 188	1 147 400	757 055
Total, Ouachita above Monroe	......	15 677	8 997 091	5 501 527
Boeuf River	28	2 221	1 003 500	240 840
Bayou Macon	29	1 366	647 500	155 400
Tensas River	30	1 542	529 500	84 720
Little River	31	3 007	1 487 000	648 927
Other drainage, Monroe to mouth of Black River	32	973	370 344	122 995
Total, Ouachita above mouth of Black River	......	24 786	13 034 935	6 754 409
Other drainage, Alexandria to mouth of Black River	33	416	247 100	107 834
Mouth of Black River to head of Atchafalaya River	34	368	85 300	13 648
Total, Red River above head of Atchafalaya River	......	91 421	33 026 477	16 746 978

Field surveys of the system shown in Fig. 1 sufficient to prepare general estimates of cost and capacity were made where existing data were insufficient. From these surveys reservoir locations were determined; dams were designed in a general way; capacities were determined for various heights of dam; the area overflowed in the reservoir was estimated; and general estimates of cost for dam and reservoir were prepared. A project for protection from future floods may be designed with all the foregoing data available.

ANALYSIS OF THE DESIGN OF A RESERVOIR SYSTEM

Assume that an area subject to flood damage has been determined; that the extent of the damage for a possible future flood is such that a considerable sum may be expended profitably in insuring against such damage; that general studies indicate considerable low-cost reservoir storage available; that

studies show that much of the flood water originates above the potential reservoir sites; and that it appears feasible to design a reservoir system, and to analyze its merits. Two general types of reservoir operation may be considered: The storage basin (Group (*A*–1)); and the retarding basin (Group (*A*–2)) mentioned previously.

Before any system is studied in detail it is necessary to decide on the measure of protection to be given. This may include protection against any future flood as great as any past floods of record, or some allowance may be made to provide for a greater future flood.

In analyzing a reservoir system particular care must be taken in considering past floods. The flood of highest crest very frequently has less volume above what might be termed a "danger stage" than some flood of smaller peak height. As the success of reservoir operation is dependent on the handling of the flood volume at the time it occurs, it is necessary to consider floods of various types. Daily hydrographs of stream flow become of great importance because the flood volume must be routed through the reservoir as it occurs and not as it might have averaged for monthly periods. Hydrographs of daily flow—at least, one covering a sharp short flood, and another a long flat flood—must be analyzed in detail. They should cover the range of experienced flows and should allow for future greater floods as may seem desirable. In some cases, hydrographs of only past floods may be considered, and the capacity of each reservoir made greater arbitrarily than that required to control these record floods in order to provide the factor of safety desired.

Having determined the degree of flood protection desired, the merits of each reservoir system should be considered and one or more methods of operation selected for detail study with the hydraulic data collected. This ordinarily means the selection of one or more design floods and the trial operation of the proposed system during such a flood.

Retarding Basin Operation.—The retarding basin method of operation (Group (*A*–2)) has gained considerable prominence in recent years largely due to its use on the Miami River, in Ohio.[3] It depends for success on the existing channel—or on an improved channel as is the case on the Miami River—being sufficient to provide for the run-off below the reservoirs and a considerable flow through the reservoirs as well. The reservoir outlet conduits are designed so that the maximum outflow, when added to the estimated local run-off below the dam sites, will not exceed the improved channel capacity. Spillways are provided so that excess flow can be released to protect the dams should the maximum stream flow exceed the designer's estimates. One of the greatest advantages of such a system is its automatic operation—no decision being required during a flood as to which gate to open and when to open it. On the other hand, it may have a limited benefit because it may so equate the flow of a tributary as to increase the flood peak of a main stream below it by changing the time of maximum contribution of the tributary.

[3] Technical Repts., Miami Conservancy Dist., Dayton, Ohio.

Considerable variation may be desirable in the size of retarding basin outlets. For an area with long moderate floods, small outlets may be desirable to give a minimum reservoir contribution to the stream below. A long duration of the flood increases the likelihood that the outflow may coincide with extreme local flows below the reservoir, and it makes a small rate desirable. For short extreme floods rather large outlets are ordinarily preferable to insure getting rid of the water as rapidly as possible. All types of intermediate designs are possible, depending on the particular flood problem.

Operation Methods for Storage Basins.—The methods of operating storage reservoirs (Group (*A*-1)) may be as follows: (1) All flow may be stored during the period determined by the rule for operation; (2) all flow may be stored except such quantities as are released uniformly: (3) all flow may be stored except a variable quantity released depending on the stream flow obtained from local areas below the reservoir; and (4) flow may be stored at a uniform rate, passing all excess flow during a calendar period.

Reservoir (1) is more nearly "fool-proof" in its operation than any other system. If based on a calendar period, or a period determined by pre-selected gauge heights, it is practically automatic except as to the release of stored waters, which must follow some predetermined rule. If the period of storage involves prediction of gauge heights to determine its time of functioning, the system becomes only as good as the prediction system.

A uniform release (Reservoir (2)) may be required by prior water rights below. It is really a minor modification of Reservoir (1). It requires somewhat less capacity to control a given flood, but the stream capacity down stream must be sufficient to contain the water released, in addition to the local inflow.

Irregular release of water (Reservoir (3)) is frequently an intricate and difficult problem. It involves knowledge of stream capacities down stream, and accurate determination of the quantity of local run-off that will reach all critical points at the same time as water released from storage. Some of this may not have fallen as rain at the time the water above the dam is released. Improper operation may result in damage suits, as many water-power operators well know. It is a complicated and uncertain method to analyze with respect to possible future floods and is usually only considered as a possible improvement in the nature of a factor of safety on a system considering the storage of all flood flows.

Reservoir (4) was considered in 1927 by the Reservoir Board for Mississippi River Flood Control.[4] This arrangement has little merit except in the case of a big system, such as the Mississippi, where a flood may come from many different sources. In the case of the Mississippi River, it has the advantage of giving a more or less dependable reduction of a definite quantity during a critical period which, if the storage system is large enough, would be sufficient to reduce the flood crest to the capacity of the existing levee system.

[4] Committee on Flood Control, Committee Doc. No. 2, 70th Cong., 1st Sess.

Storage systems (Group (*A*–1)) are necessary wherever the local run-off below the reservoir may require most of the available channel capacity. Reservoirs of this type have the advantage of being capable of controlling stream flow as desired. It is possible to reduce the flow from the reservoir at will. However, once it is stored, it is frequently a serious problem to determine just when and how to let it go.

Operating the Reservoir.—The storage or retarding of flow during the flood-danger period must be carefully analyzed before selecting the type of operation to adopt. Most streams have many rises or crests. Most of these rises do no damage to the valley. Only certain rises become high enough to cause damage. As the rain falls and gauge readings begin to rise the question looms up: "Will this be a damaging flood, and if so, how much of it can be passed safely down the river before the reservoir gates are closed?" In certain cases it is possible to store every flood at the reservoir as it occurs, but in most cases the storage capacity is limited either by physical features or cost, and it is necessary to permit all small floods to pass. In the case of the retarding basin, the size of outlets must be determined in the design. In the case of the storage basin not only must outlets be designed, but a rule must be selected for their operation.

It is of prime importance that the rule of operation for a storage system be determined in advance, and be such that it can be used progressively as the flood develops. It is very simple to design a reservoir system to control a flood peak that has occurred in the past, but foresight is limited during a flood. Certain information is obtained from gauges above the reservoirs and, in many cases, flows may be predicted for limited periods in advance. With these data, the rule of operation must be workable. The entire flood is not visible to the operater, and he must do his work on the basis of what he can see.

In cases where the period of operation can be based on definite stages at key gauges, its determination is quite simple. A study of past floods shows that all flow above a given stage must be stored or regulated. When this stage is reached the reservoirs are placed in operation.

Operation on a rule requiring prediction of gauge heights is frequently desirable. If all flows above a fixed stage are stored, large reservoir capacity is required. This may prove costly and it becomes desirable to utilize storage as nearly as possible to impound only that part of the flood volume that causes the damage. Water stored before the danger stage is of no value in reducing floods and merely fills space required by the flood water that will cause damage.

The most common method of predicting advance stages is by studying the relation between comparable gauges. Where no important tributary or no large storage basin is located between the gauges compared, this is a fairly satisfactory method. However, even when these conditions do not exist, changes in relation may occur from time to time due to changes in river regimen. Fig. 5 gives an illustration of a typical gauge-relation study for a case that has no complicating factors. This shows the relation between

gauges at Columbus, Ky., and New Madrid, Mo., on the Mississippi River. With an experienced stage at the up-stream station the probable stage at the lower station may be read directly from the curve. In cases where it is necessary to extend past a tributary, the volumetric contribution of the tribu-

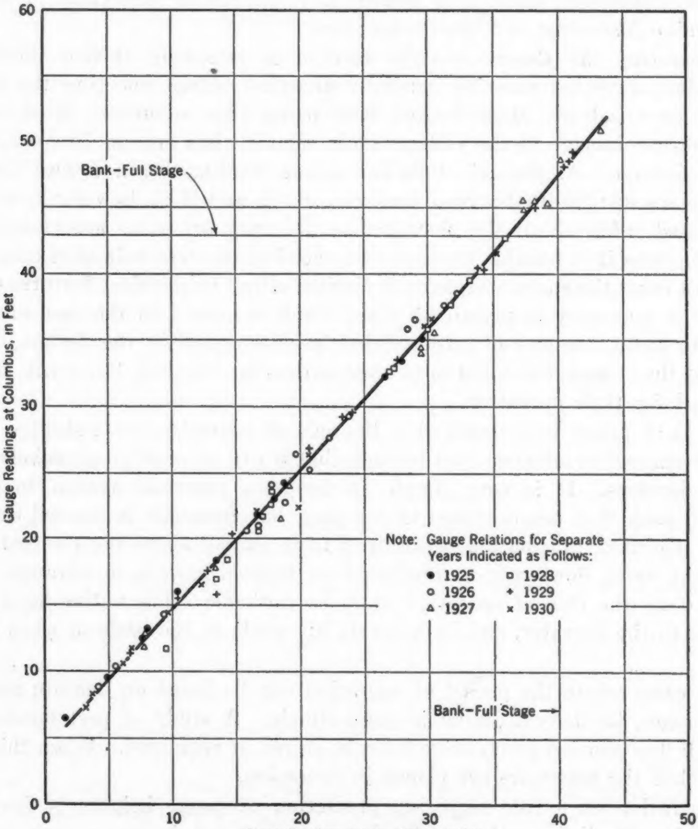


FIG. 5.—RELATION BETWEEN GAUGES, COLUMBUS, KENTUCKY, AND NEW MADRID, MISSOURI, MISSISSIPPI RIVER, 1925–1930

tary coinciding with the crest of the main stream may be estimated and applied as a correction to the main river gauge below. Similarly, an allowance may be made for a storage basin. The results are more or less approximate. On many streams fairly good results may be obtained. It is estimated that stages for Cairo, Ill., at the mouth of the Ohio River, may be predicted at least four or five days in advance and that stages at New Orleans, on the Mississippi River, may be predicted about two weeks or more in advance. Under conditions of small contributions of lower tributaries these periods may be lengthened somewhat. However, on many small streams, the forward limit of predictions may be a matter of hours.

A single-key gauge at or near a critical area with a definite maximum allowable stage may be used as a basis for operating reservoirs within prediction distance of this gauge. For instance, gauge heights may be predicted at Cairo, Ill., five days in advance. If 50 ft is set as an allowable maximum at this gauge, all reservoirs within a water-flow range of five days may be operated to store their entire contribution at all times, when 50 ft, or more, seems probable. By allowing a few feet as a factor of safety (it is advisable to try to hold to 50 ft when, say, 53 ft will still do no damage), this system can be made fairly satisfactory. Undoubtedly, it gives effective use of close-in storage. A reservoir five days away, for example, will begin storing the day a gauge reading of 50 is predicted (predictions made five days ahead). The reservoir one day away would not begin storing until the day before a gauge reading of 50 ft was predicted. In this way the maximum quantity of harmless (less than 50 ft) water would be permitted to pass down stream and the near-by reservoirs would be held in readiness to make their storage most effective in impounding water only when it would cause damage.

An extension of the key-gauge method is sometimes possible on a large system. It may be found that a combination of certain maximum gauge heights on a group of tributaries will not give trouble. Any height in excess of that stated for any tributary may give trouble; therefore, a key gauge may be picked for each tributary and each one regulated so that its flow will not exceed its allowable maximum. By preventing any tributary from introducing a damaging quantity of water, the flow of the main river may be kept at less than its own allowable maximum at the danger zone.

It is sometimes found that a key gauge on the main stream may be selected and all flow that originates above reservoirs for a given stage at the key point, when something less than a higher stated crest is predicted, may be passed safely through the danger zone. For instance, if Arkansas City is the key gauge on the Mississippi River for the operation of reservoirs on the Red River that protect the Mississippi River against Red River floods, it is found by studying gauge records that when not more than 50 ft is predicted at Arkansas City, no water need be stored in Red River reservoirs; but if a gauge of more than 50 ft is predicted, storage should begin at 35 ft at all reservoirs 28 days from the mouth of Red River and at corresponding dates for closer reservoirs. Storage of all flow at reservoirs should continue until 48 ft is reached on a falling stage after the predicted crest. This method, checked against 40 or 50 years of hydrographs, was found to be reliable in all cases.

In many cases no satisfactory stage-predicting system can be developed. It is necessary then to store all flow above reservoirs for a calendar period and to equate all floods to a fixed maximum contribution of the stream during the remainder of the year and during the period in which a reservoir is emptying. This requires a large storage, and there is usually considerable unused storage in any single flood.

On some streams an historic study will show that all damaging floods of record have occurred during a comparatively short calendar period. Analysis

of conditions causing these floods may show that weather conditions are such that, barring a change of climate, the same experience may be expected in the future. Under such conditions the rule of operation may be based on a specified operation during the dangerous calendar period.

Tests of Operation Method Selected.—Having selected the method or methods of reservoir operation considered best adapted to the solution of a particular flood problem, each method must be tested by determining its operation during the selected type floods of record and the design flood. In making these tests it is important that the operation be based only upon the information that would be available each day as the flood actually developed. Only the foresight of tested prediction can be considered.

A retarding-basin system of reservoirs (Group (*A*-2)) has been selected to demonstrate the foregoing points. One of the reservoirs in the system under consideration is near Antlers, Okla., on the Kiamichi River. Fig. 6

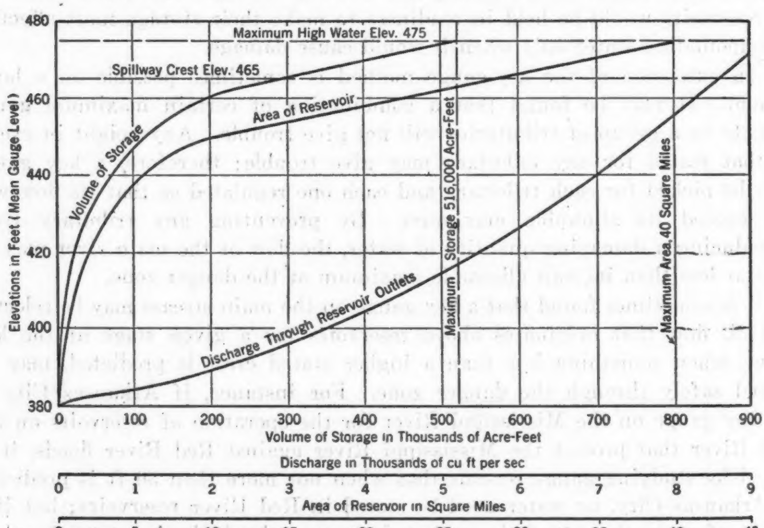


FIG. 6.—AREA AND CAPACITY CURVES, PROPOSED RESERVOIR, NEAR ANTLERS, OKLAHOMA

shows the storage for various elevations of water surface and the discharge through the reservoir outlets. A general summary of pertinent design data for a masonry dam, in this connection, is as follows:

Drainage area above the reservoir, in square miles......	1400
Surface area of proposed reservoir, in acres...........	25600
Storage capacity, in acre-feet..........................	516000
Height of dam above the valley, in feet..............	105
Length of dam at crest, in feet........................	950
Elevation, crest of dam, in feet.......................	485
Elevation, crest of spillway, in feet..................	465
Cost of reservoir, in dollars per acre-feet of storage.....	6.26

TABLE 3.—RESERVOIR OUTFLOW CURVE, ANTLERS RESERVOIR, KIAMICHI RIVER

Difference in water surface, reservoir and river below, in feet (1)	Computed discharge through conduits, in cubic feet per second (2)	Equivalent gauge height at Belzoni gauge, in feet (3)	Stage above low water required to pass conduit discharge (river below reservoir), in feet (4)	Water-surface elevation in reservoir to pass conduit discharge, in feet (mean Gulf level) (5)
0.5	660	6.1	2.6	383.1
1.0	940	6.7	3.2	384.2
2.0	1 340	7.4	3.9	385.9
3.0	1 660	8.0	4.5	387.5
5.0	2 180	9.0	5.5	390.5
7.0	2 600	9.6	6.1	393.1
10.0	3 120	10.6	7.1	397.1
15.0	3 880	11.8	8.3	403.3
20.0	4 480	12.8	9.3	409.3
30.0	5 560	14.6	11.1	421.1
45.0	6 840	16.5	13.0	438.0
60.0	8 000	18.5	15.0	455.0
80.0	9 260	20.3	16.8	476.8
100.0	10 440	21.8	18.3	498.3

Table 3 gives the information from which the outflow curve was determined. Fig. 7 is a hydrograph of the unmodified 1927 flood at the reservoir site. In Column (2), Table 3, ten conduits, 5 ft in diameter, were considered,

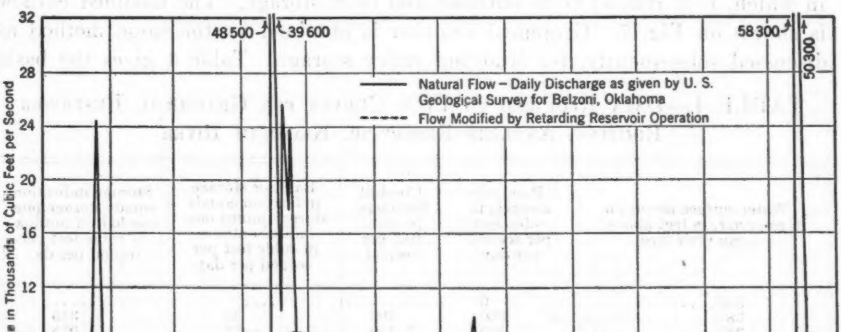


FIG. 7.—DISCHARGE HYDROGRAPH, KIAMICHI RIVER, AT ANTLERS RESERVOIR

and the discharge was computed by the Hazen-Williams formula, assuming entrance and exit losses equal to velocity head; that is,

$$H = H_f + H_o \text{.........................} (1)$$

With $H_f = L\,S$ and $H_o = \frac{V^2}{2g}$;

$$V = 1.318\, C_1\, R^{0.63}\, S^{0.54} \text{.........} (2)$$

in which, H = the total head available (Column (1), Table 3); H_f, the head consumed in friction; H_o, the head loss at entrance and exit, assumed

equal to velocity head; L, length of conduits; $S = \frac{H_f}{L}$; V, mean velocity in conduits; C_1, roughness coefficients of conduits; and R, hydraulic radius of conduits.

The Belzoni gauge referred to in Column (3), Table 3, is a regular station situated a short distance below the reservoir. Low water is 3.5 ft above zero on this gauge. In Column (4), it is assumed that the relative stage discharge is approximately the same below the reservoir as it is at the Belzoni gauge. Referring to Column (5), low water on the zero stage in the reservoir is 380 ft above mean Gulf level. The water surface in the reservoir is equal to Elevation 380 plus the river stage below the reservoir plus the difference in the water surface required to give the various discharges.

To determine the effect of the Antlers Reservoir on the 1927 flood (one of the large record floods), the inflows have been routed through the reservoir and the modified outflow has been determined. The routing may be accomplished by either analytical or graphical solution of the equation:

$$I = O + S \quad \text{.................. ....(3)}$$

in which, I = inflow; O = outflow; and S = storage. The modified outflow is shown on Fig. 7. Graphical solution is obtained by the same method as discussed subsequently, for studying valley storage. Table 4 gives the basic

TABLE 4.—DATA REQUIRED TO PLOT CURVES FOR GRAPHICAL RESERVOIR ROUTING, ANTLERS RESERVOIR, KIAMICHI RIVER

Water-surface elevation in reservoir, in feet above mean Gulf level	Reservoir storage, in cubic feet per second per day	Conduit discharge, in cubic feet per second	Base for storage indication equals storage minus one-half of outflow, in cubic feet per second per day	Storage indication equals storage plus one-half of outflow, in cubic feet per second per day
380	0	....	......	....
382	200	290	55	345
385	500	1 150	—75	1 075
388	800	1 800	—100	1 700
390	1 000	2 140	—70	2 070
395	1 850	2 860	420	3 280
400	3 000	3 500	1 250	4 750
410	6 500	4 500	4 250	8 750
420	12 500	5 450	9 775	15 225
430	21 500	6 250	18 375	24 625
440	37 000	7 000	33 500	40 500
450	60 000	7 650	56 175	63 825
460	98 500	8 300	94 350	102 650
465	150 000	8 600	145 700	154 300
475	258 000	9 200	253 400	262 600

data used in plotting the curves for this particular routing. Different scale curves were plotted for the different ranges in storage to make the accuracy of the graphical work comparable to the accuracy of the flows being routed. Table 5 is an extract from the graphical routing. The curves used were similar to those in Fig. 8 used for the studies of valley storage effect by the "reach" reservoir method. The method of using the curves is discussed subsequently. Analytical routing requires only the use of storage and outflow curves with inflows. Table 6 gives the analytical routing of the same inflows as that cov-

TABLE 5.—GRAPHICAL RESERVOIR ROUTING, ANTLERS RESERVOIR, KIAMICHI RIVER

Date	Inflow, in cubic feet per second	Inflow sum, in cubic feet per second	Average inflow, in cubic feet per second	Accumulated storage, in cubic feet per second per day*	Outflow, in cubic feet per second†	Water-surface elevation in reservoir, in feet above mean Gulf level‡
1927:						
July 14	39	....	......	200	39	382.0
July 15	8 510	8 549	4 275	2 600	3 270	398.2
July 16	10 100	18 610	9 305	7 900	4 770	412.7
July 17	3 940	14 040	7 020	10 000	5 120	416.3
July 18	1 160	5 100	2 550	7 650	4 720	412.2
July 19	617	1 777	888	4 200	3 930	404.0
July 20	436	1 053	526	1 450	2 580	393.0
July 21	330	766	383	280	520	382.8
July 22	265	595	297	200	290	382.0
December 11	78	....	......	200	78	382.0
December 12	465	543	271	200	271	382.0
December 13	26 100	26 565	13 282	10 200	5 200	417.0
December 14	58 300	84 400	42 200	47 000	7 320	445.0
December 15	50 300	108 600	54 300	93 000	8 200	458.7
December 16	24 400	74 700	37 350	121 000	8 440	462.3
December 17	4 260	28 660	14 330	127 000	8 470	462.9
December 18	1 700	5 960	2 980	122 300	8 440	462.4
December 19	1 280	2 980	1 490	115 700	8 410	461.8

*As scaled from storage curve. † As scaled from outflow curve. ‡ After first day, elevation is determined by storage indication curve.

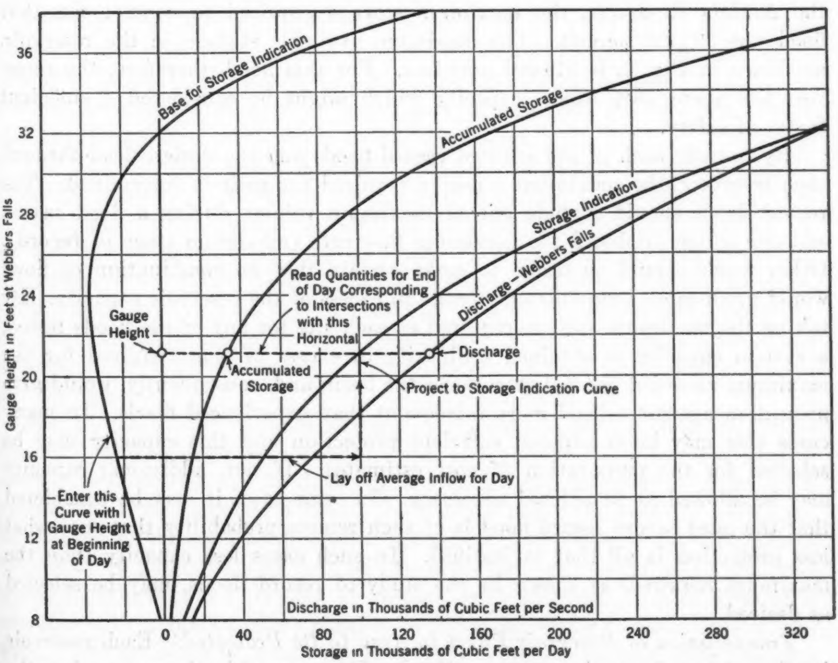


FIG. 8.—REACH RESERVOIR ROUTING, ARKANSAS RIVER, TULSA TO WEBBERS FALLS, OKLAHOMA

TABLE 6.—ANALYTICAL RESERVOIR ROUTING, ANTLERS RESERVOIR, KIAMICHI RIVER

Date	Inflow, in in cubic feet per second	Average inflow, in cubic feet per second	Storage change, in cubic feet per second per day	Accumulated storage, in cubic feet per second per day	Outflow, in cubic feet per second	Average outflow, in cubic feet per second	Water surface elevation in reservoir, in feet, mean Gulf level
1927:							
July 14	39			200	39		382.0
July 15	8 510	4 275	2 600	2 800	3 360	1 700	399.0
July 16	10 100	9 305	5 300	8 100	4 760	4 060	412.5
July 17	3 940	7 020	2 100	10 200	5 100	4 930	416.0
July 18	1 160	2 550	—2 400	7 800	4 680	4 890	411.5
July 19	617	888	—3 400	4 400	3 920	4 300	404.0
July 20	436	526	—2 700	1 700	2 600	3 260	393.0
July 21	330	383	—1 350	350	750	1 670	383.5
July 22	265	297	—150	200	290	520	382.0
December 11	78			200	78		382.0
December 12	465	271	30	230	380	229	382.3
December 13	26 100	13 282	10 470	10 700	5 220	2 800	417.5
December 14	58 300	42 200	36 000	46 700	7 310	6 265	444.7
December 15	50 300	54 300	46 600	93 300	8 200	7 755	458.8
December 16	24 400	37 350	29 000	122 300	8 440	8 320	462.4
December 17	4 260	14 330	5 900	128 200	8 480	8 460	463.0
December 18	1 700	2 980	—5 500	122 700	8 450	8 465	462.5
December 19	1 280	1 490	—7 000	115 700	8 410	8 430	461.8

ered by the graphical routing in Table 5. In addition to the outflow, the routing study gives the accumulated storage in the reservoir each day. For the Antlers Reservoir, the maximum storage required to control the 1927 flood was 393 000 acre-ft. The maximum available storage in the reservoir, as shown in Fig. 6, is 516 000 acre-feet. For this flood, therefore, the reservoir has about 25% excess capacity which might be considered a sufficient factor of safety.

By routing each of the selected record floods and the design flood through each reservoir, the maximum capacity required for each is determined. The record floods should include one of maximum volume during a flood season and one which produced the maximum flow rate (maximum stage of record). Other floods should be tested to make certain that no combination of flows would produce an accumulated storage in excess of the reservoir capacity. By taking the maximum storage required at each site for any of the floods tested, a system capacity is obtained ordinarily in excess of that required for the maximum recorded general storm over the basin and, consequently, would give protection against a flood more infrequent than experienced floods. In many cases this may be considered sufficient protection and this capacity may be selected for the preparation of cost estimates. If not, additional capacity may be allowed as considered necessary. In some cases it may be considered that the most severe record flood is of such remote probability that somewhat less protection is all that is justified. In such cases less capacity than the maximum required, as shown by the study of record floods, may be selected as desired.

Transmission of Reservoir Effect to Area to Be Protected.—Each reservoir in the system is analyzed as outlined. However, this does not give the controlled flood at the critical area unless perchance there is only one reser-

voir and the area is directly below it. In other cases, the controlled outflow of the reservoirs must be followed down the stream channels below, increased by the run-off below the dam, combined with the flow of other tributaries whether or not controlled by reservoirs, and the modifying effect of the channel or valley storage on the new flood wave determined. By comparing this wave with the unmodified flow the reduction at the area to be protected can be determined and the success of the project measured.

Determining the modified flood wave involves three major problems: (*A*) Time travel of reduced wave from reservoir to critical area; (*B*) run-off of local area below reservoirs; and (*C*) effect of "valley storage." By combining reservoir outflows, allowing for differences in time of travel, adding local run-off, and allowing for the effect of valley storage, controlled flow at key points in the critical area is determined.

The time of travel of flood crests (Problem (*A*)) is difficult to determine. It varies considerably depending on the height of the flood wave and its position relative to previous floods. In many cases the waves become merged with preceding crests, and it is impossible to identify any particular crest at the lower station. The most satisfactory method of determining time of travel is to plot the time between two stations as a function of stage. Even at best, considerable approximation is necessary to arrive at a schedule of time travel.

The run-off of local areas (Problem (*B*)) can be determined ordinarily by proportioning the drainage area to some adjacent area for which the run-off is known and computing run-off as proportional to drainage area. In some cases it is necessary to approximate run-off from rainfall records. This is not particularly satisfactory as daily flows must be estimated and rainfall run-off relations do not give particularly satisfactory daily flows.

The inflow to successive reaches of the main stream may be determined by adding reservoir outflows to local run-off (Problem (*C*)). If the channel or over-bank valley storage is fairly large these flows will be flattened or equated in passing through a given reach. Two methods of computing this flattening are presented. The second is in fact only a general approximation, but in many cases it is all that is possible. The many uncertainties affecting computations often make exact determination of valley storage effects impracticable, and allowances by judgment of the designing engineer are all that are feasible.

The two methods presented for computing valley storage effect might be termed: (*a*) Reach-reservoir method; and, (*b*) proportioning to hydrographs plotted from experience. Method (*a*) has been used in various forms at a number of U. S. Engineer Offices. Each office using the method has introduced some special modifications, but the general principles are approximately the same. In principle, this method considers the river length to be analyzed as being divided into a number of reaches, each of which is considered as a reservoir with a surface slope parallel to the mean water surface slope of the reach. The quantity of water stored in this reach for various gauge heights at the lower end is determined by computing the day-by-day storage from Equation (3).

If the daily inflow and outflow are known for a given flood, the daily storage changes may be computed from Equation (3). Summing the daily changes, the accumulated storage at successive dates may be computed and plotted as a function of the gauge height at the lower end of the reach (using the same gauge for which the discharge is also known). The plotted storages will follow approximately a uniform curve varying somewhat with rising and falling stages, but ordinarily sufficiently close so that an average curve may be drawn through them.

Having determined a storage curve for the reach for a known flood or series of floods and knowing the stage-discharge relation, other floods for which only inflows are known, may be routed through the reach, and its storage effect on them determined. In its simplest form this would consist of entering the reach with the inflows of the flood to be tested, modifying these flows by the storage, and determining the outflow. This routing can be accomplished graphically by curves such as those in Fig. 8. These consist of a storage curve; an outflow curve; a base for a storage indication curve (storage minus one-half the outflow); and, a storage indication curve (storage plus one-half the outflow).

Fig. 8 demonstrates the method of using these curves to obtain stage, outflow, and an accumulated storage at the end of each day for a known stage at the beginning of the day, and inflow during the day. To use the curves, beginning stages and beginning inflows are required. Mean values are obtained by averaging the inflow of one day with that of the following day. The graphical solution gives outflow at the end of the day corresponding to the stage at the same time. Averaging this outflow with that for the preceding day and adding the storage change, the total checks the inflow. The construction of the curves is such as to conform to these requirements.

In applying the reach-reservoir method, it is found that the inaccuracies of gauge and discharge data make certain modifications advisable. For instance, when the storage curve is plotted it only passes through the average of the points used for its construction. If a study is being made of the reduction that can be obtained on a certain flood of record by the operation of certain reservoirs to withhold known quantities during a flood period, what is called a differential routing study is made. First, the actual stages obtained during the unmodified flood wave are tabulated. Using these stages and the known outflows, the inflows as they would scale using the storage curve, are determined. Next, inflow reductions from the river reach above and from reservoirs in the reach are determined. These are subtracted from the adjusted inflows (those scaled using the storage curve) and reduced inflows are determined for use with the routing curves. These curves give reduced stages and outflows such as would result from the reach-reservoir operation. Entering the next reach, the outflow reductions are used. As these are differentials from the actual outflows computed, using inflows that checked the storage curve, they are more nearly correct than if the entire inflow and outflow were carried through the routing. An example of this differential routing is given in Table 7 for a part of the 1927 flood routed through the

TABLE 7.—REACH-RESERVOIR METHOD, FLOOD ROUTING, ARKANSAS RIVER, TULSA TO WEBBER'S FALLS, OKLAHOMA, VALLEY STORAGE STUDY

Date	Daily reduction in flow at Tulsa (head of reach), in cubic feet per second	DAILY REDUCTIONS IN FLOW FROM:		Total daily reduction in reach, in cubic feet per second	Mean daily reduction, in cubic feet per second	Mean natural daily inflow adjusted to storage curve, in cubic feet per second	Mean modified daily inflow, in cubic feet per second	FOOT OF REACH, WEBBER'S FALLS				
		Oologah Reservoir on Verdigris River, in cubic feet per second	Pensacola, Markham, and Fort Gibson Reservoirs on Grand River, in cubic feet per second					Modified stage, in feet	Modified daily outflow, in cubic feet per second	Natural stage, in feet	Natural daily outflow, in cubic feet per second	Daily reduction in outflow, in cubic feet per second
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(3)	(9)	(10)	(11)	(12)
1926:												
April 10	1 200	....	....	1 200	600	125 000	124 400	20.2	126 600	20.2	126 600	....
April 11	2 100	....	43 000	45 100	22 500	130 000	107 500	18.8	107 700	20.7	133 300	25 600
April 12	2 300	....	78 000	80 300	62 700	174 000	111 300	19.4	116 400	23.8	179 600	63 200
April 13	19 200	12 900	109 000	141 100	110 700	244 000	133 300	21.1	139 100	27.3	241 100	102 000
April 14	31 600	17 800	181 000	230 400	185 700	354 000	168 300	23.4	173 400	31.6	330 800	157 400
April 15	18 700	32 800	210 000	261 500	245 900	387 000	141 100	21.4	143 300	33.1	365 400	222 100
April 16	6 400	29 100	197 000	232 500	247 000	343 000	96 000	17.3	89 600	32.5	350 500	260 900
April 17	3 500	35 200	185 000	223 700	228 600	297 000	68 400	14.7	61 400	31.0	317 000	255 600
April 18	1 500	55 000	188 000	244 500	234 100	307 000	72 900	16.2	76 900	30.8	313 000	236 100
April 19	800	39 800	166 000	206 600	225 500	312 000	86 500	17.4	90 800	30.9	315 000	224 200
April 20	7 800	38 300	132 000	178 100	192 300	273 000	80 700	16.3	78 000	29.6	287 700	209 700
April 21	7 600	25 600	129 000	162 200	170 100	260 000	89 900	17.9	96 500	28.7	268 900	172 400
April 22	4 700	27 400	32 000	64 100	113 200	275 000	161 800	23.4	173 400	29.1	277 300	103 900
April 23	2 100	23 200	....	25 300	44 700	244 000	199 300	25.1	200 800	27.9	252 600	51 800
April 24	1 800	14 600	....	16 400	20 800	194 000	173 200	23.7	178 000	25.2	202 600	24 600
April 25	600	10 500	....	11 100	13 800	164 000	150 200	22.1	153 500	23.1	168 600	15 100
April 26	600	....	....	600	5 800	147 000	141 200	21.4	143 300	21.8	149 100	5 800
April 27	......	....	....	....	300	143 000	142 700	21.6	146 200	21.6	146 200	....

reach of the Arkansas River from Tulsa to Webber's Falls, Okla. The inflow reduction from the reach above is entered in Column (1), Table 7, and the outflow reduction from the reach is given in Column (12). This method was used with considerable success by the Memphis (Tenn.), Engineer District in studying flood control of the White and Arkansas Rivers.

The other method of determining valley storage effect (Method (*b*)) is to plot a large number of flood-flow hydrographs for the gauges at the head and foot of a reach. It is usually found that one of these hydrographs closely approximates the hydrograph of inflows to the reach as modified by the reservoir system. By the simple expedient of modifying the computed outflow hydrograph from the reach to correspond to the effect obtained on the hydrograph based on experience, the approximate effect of valley storage is determined. This is admittedly a more or less approximate solution, but in many cases it is the only feasible method and gives fair results if the hydrograph used as a model is rather closely approximated by the hydrograph being analyzed.

After computing effects of valley storage and increase due to local run-off below reservoirs, the controlled flow at the key gauges at the overflow area is determined for the reservoir system being analyzed for each of the various test floods. By use of a stage-discharge relation at each of these gauges the stage as it would be controlled is determined. If each of the record floods

and the design flood result in controlled stages below bank-full at the key points when routed through the selected retarding-basin reservoir system, the system will protect the danger area.

Any of the storage systems described may be tested in a manner similar to the retarding-basin system. Reservoirs are selected—design storms determined—and a rule of operation is adopted. Using the design inflow and following the selected rule of operation, the modified flow and the storage required are computed. If the modified flow results in gauge heights below the danger stage at the overflow area for each flood and if the storage required is less than the reservoir capacity, the system will be satisfactory.

Hydrographs of the natural and modified flow at the reservoir site are shown in Fig. 9 for a theoretical reservoir on the Gasconade River, at Rich

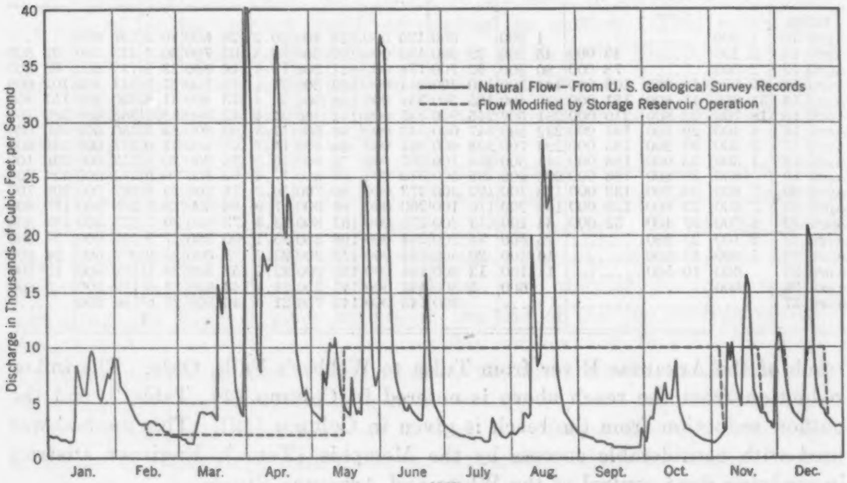


FIG. 9.—DISCHARGE HYDROGRAPH, GASCONADE RIVER, AT RICH FOUNTAIN, MISSOURI

Fountain, Mo. It has been assumed that this reservoir would operate to store all flow except a uniform release of 2 000 cu ft per sec for the period from February 15 to May 15. During the remainder of the year it would regulate the flow to a maximum of 10 000 cu ft per sec. Typical extracts for the operation during 1927 are shown in Table 8(*a*). During the period from February 15 to May 15 all flow in excess of 2 000 cu ft per sec has been stored. Several fairly large flood peaks have been held back and the flow of the tributary has been controlled to the fixed release value of 2 000 cu ft per sec. A storage of 1 467 100 acre-ft would have been required. The effect at a down-stream point would be determined by considering the inflow reductions as they would have been modified by valley storage. To continue the operation to regulate the flow for the remainder of the year to 10 000 cu ft per sec a maximum rate would have required a capacity of 1 694 560 acre-ft.

Hydrographs of the natural and modified flow at the reservoir site are shown in Fig. 10 for a theoretical reservoir on the Kansas River, at Bonner

TABLE 8.—Results of Storage Reservoir Operation (Given Only in Typical Periods)

Date	Inflow, in cubic feet per second	Storage, in cubic feet per second per day	Accumulated storage, in cubic feet per second per day	Outflow, in cubic feet per second
(*a*) Gasconade River at Rich Fountain, Mo., January 1 to February 14, Outflow Equals Inflow				
1927:				
February 15	2 930	930	930	2 000
February 16	3 290	1 290	2 220	2 000
February 17	3 290	1 290	3 510	2 000
February 18	3 170	1 170	4 680	2 000
February 25	2 110	110	8 810	2 000
February 26	2 110	110	8 920	2 000
February 27	2 000	0	8 920	2 000
February 28	1 890	—110	8 810	2 000
March 18	3 290	1 290	24 690	2 000
March 19	15 700	13 700	38 390	2 000
March 20	14 500	12 500	50 890	2 000
March 21	13 100	11 100	61 990	2 000
May 13	9 160	7 160	726 360	2 000
May 14	6 160	4 160	730 520	2 000
May 15	5 030	3 030	733 550	2 000
May 16	4 380	—5 620	727 930	10 000
May 17	4 010	—5 990	721 940	10 000
May 18	3 770	—6 230	715 710	10 000
June 6	18 900	8 900	846 780	10 000
June 7	10 500	500	847 280*	10 000
June 8	7 780	—2 220	845 060	10 000
(*b*) Kansas River at Bonner Springs, Kans., January 1 to February 28, Outflow Equals Inflow				
1927:				
March 1	2 500	2 500	2 500	0
March 2	2 660	2 660	5 160	0
March 3	2 660	2 660	7 820	0
March 31	3 180	3 180	94 240	0
April 1	28 700	28 700	122 940	0
April 2	47 200	42 060	165 000	5 140
April 3	26 100	5 000	170 000	21 000
April 4	17 000	5 000	175 000	12 000
May 29	5 880	5 000	450 000	880
May 30	5 150	5 000	455 000	150
May 31	4 670	4 670	459 670	0
June 1	4 010	—15 990	443 680	20 000
June 2	4 670	—15 330	428 350	20 000
June 25	20 800	800	461 400	20 000
June 26	20 400	400	461 800†	20 000
June 27	18 700	—1 300	460 500	20 000
June 28	16 800	—3 200	457 300	20 000
August 4	5 880	—14 120	18 320	20 000
August 5	7 150	—12 850	5 470	20 000
August 6	10 100	—5 470	0	15 570
August 7	8 200	0	0	8 200
(*c*) Osage River at Bagnell, Mo., January 1 to March 15, Outflow Equals Inflow				
1927:				
March 16	8 590	8 590	8 590	0
March 17	10 000	10 000	18 590	0
March 18	11 600	11 600	30 190	0
April 16	103 000	103 000	2 012 090	0
April 17	106 000	106 000	2 118 090	0
April 18	105 000	105 000	2 223 090	0
April 19	102 000	102 000	2 325 090	0
April 20	102 000	102 000	2 427 090	0
April 25	84 500	84 500	2 893 590	0
April 26	79 700	79 700	2 973 290‡	0
April 27	73 100	0	2 973 290	73 100
April 28	66 300	0	2 973 290	66 300
May 12	55 400	0	2 973 290	55 400
May 13	48 100	—26 900	2 946 390	75 000
May 14	37 900	—37 100	2 909 290	75 000
June 5	80 400	80 400	2 611 980	0
June 6	58 200	58 200	2 670 180	0
June 7	40 000	0	2 670 180	40 000
July 29	6 760	—68 240	55 450	75 000
July 30	6 000	—55 450	0	61 450
July 31	6 500	0	0	6 500

* Maximum storage equals 1 694 560 acre-ft. † Maximum storage equals 923 600 acre-ft.
‡ Maximum storage equals 5 946 580 acre-ft.

Springs, Kans. It has been assumed that the reservoir would operate so as to store at a uniform rate of 5 000 cu ft per sec per day when such flow was available during the period from March 1 to June 1. If a deficiency from the

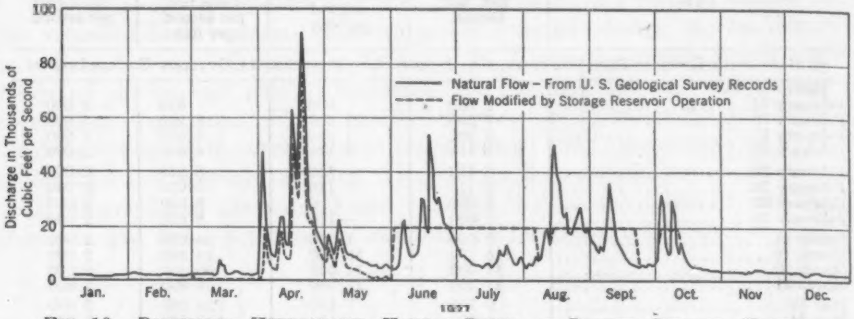


FIG. 10.—DISCHARGE HYDROGRAPH, KANSAS RIVER, AT BONNER SPRINGS, KANSAS

schedule develops, this deficiency may be compensated by the first rise that will make it up. Table 8(*b*) shows typical extracts from the reservoir operation. The reservoir could be counted on to reduce flows at down-stream points by a uniform quantity of 5 000 cu ft per sec, except for the first month when the flow was less than this amount. A maximum capacity of 919 340 acre-ft would have been required to accomplish this between March 1 and June 1. It has further been assumed that during the remainder of the year, the reservoir would operate to control the maximum peak flow of the stream to 20 000 cu ft per sec. This operation would require a peak storage of 923 600 acre-ft.

Hydrographs of the natural and modified flow at the reservoir site are shown in Fig. 11 for a theoretical reservoir on the Osage River, at Bagnell,

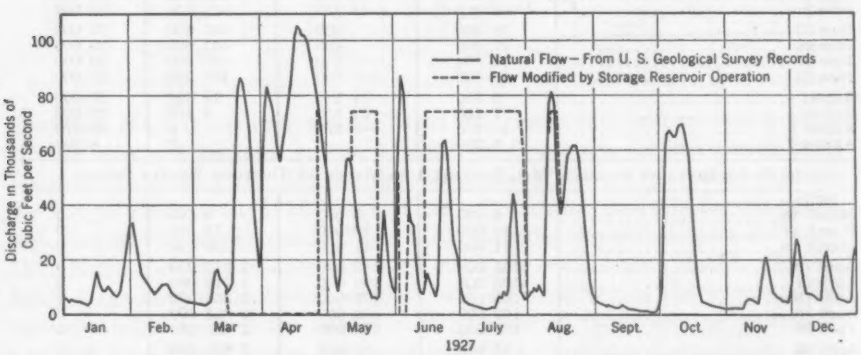


FIG. 11.—DISCHARGE HYDROGRAPH, OSAGE RIVER, AT BAGNELL, MISSOURI

Mo. This reservoir is assumed as being five days from Cairo, Ill., and operated to store all flow when 50 ft or more is predicted on the Cairo gauge. It is permitted to release water at a rate not to exceed 75 000 cu ft per sec, when 40 ft or less is predicted at Cairo. Extracts of the operation for 1927 are shown in Table 8(*c*). It is noted that the reservoir stored at a nearly uni-

form rate of about 100 000 cu ft per sec. for several days before, during, and after the Cairo crest. Even allowing for valley storage reduction this would mean practically 100 000 cu ft per sec could have been withheld from the Cairo flood crest by this reservoir. To have accomplished this would have required a storage of slightly less than 6 000 000 acre-ft. The storage required to accomplish the regulation for the remainder of the year would be less than this quantity.

Comparison of Systems.—The result that might be obtained by each of several different systems should be carefully compared and analyzed. The particular advantages of each should be weighed and balanced against the disadvantages. The cost of each system should be determined and its annual charges computed. The cost and difficulties of maintenance, operation, and renewal should be considered. By comparing costs (including annual charges) with the results obtained, the best system for any particular project may be selected. If the cost is less than the estimated value of protection, the plan may be justified, providing the cost of financing is not so great as to reverse this balance. To be adopted, however, not only must these conditions exist, but the plan must be cheaper than any other plan that will give equivalent satisfactory protection.

Combined Use of Reservoirs

Reservoirs acting alone as flood-control structures have been treated thus far in this paper. Extension of the function of the reservoir to include other interests in combination with flood control is sometimes feasible.

General Classification.—The consideration of combined use of a reservoir for several purposes may be divided logically into two general classifications: (*A*) Combined-purpose storage, or joint use of the same storage space for different purposes; and (*B*) combined-interval storage, or separate storage units combined in one reservoir to reduce cost. Each class should be considered for: (*a*) Combined operation to provide flood control and to obtain development of water power; (*b*) flood control and irrigation; (*c*) flood control and domestic or industrial water supply; (*d*) flood control and navigation; and (*e*) combination of any three or all of the purposes mentioned in Items (*a*) to (*d*).

Flood Control and Water Power.—Usually, the possibility of obtaining water power from a reservoir constructed for flood protection (Class (*A-a*)) is considered hopefully by every organization planning a flood-control system. It seems to offer a solution to that perplexing problem of how to pay for the flood-protection plan. A water-power plant might furnish revenue to pay operating expenses and to retire bonds. However, detailed consideration ordinarily reveals many inherent difficulties to such a plan.

In an ordinary retarding-basin plan, where it is not possible to utilize the lower part of the reservoir as a retention basin or "head developer," there will be considerable periods when no head is developed at the dam and, consequently, no power. This would mean a large power-plant installation for only brief and uncertain periods of operation. Only a very general considera-

tion is necessary to satisfy the engineer that no saving in cost can be obtained by including a power plant in such a plan.

For a storage system there would also be no head during periods of no storage. However, the lower part of the reservoir often develops considerable head for only a small percentage of the total capacity. By considering this lower part as below the draw-down limit and used only to develop head, it is possible to have a minimum head for power development. This storage volume is considered non-effective and is not included in determining the required capacity.

The demands of water power and flood control are more or less contrary. The power reservoir is filled as rapidly as possible during periods of high flow to insure a maximum yearly output. When the reservoir is full and high flows occur they are merely passed over the spillway. On the other hand, the flood-control reservoir is kept as low as possible, in order that the storage capacity may be available to store the peak of the abnormally high floods. A large power reservoir is frequently a very efficient reducer of small floods, but it may have little or no effect on large floods should they occur at the time of year when the reservoir is nearly full. A flood-control storage reservoir, even with a minimum draw-down limit, would permit of very irregular development of water power. There would be practically run-of-river operation through the minimum head during long periods of ordinary flow when it is desired to keep the reservoir low. There might be little or no development during the storage period, depending on the quantity of water that could be released under the method of operation adopted. During the emptying period after the flood, fairly efficient, but still variable, power development might be obtained.

In a very large interconnected power system, it is possible that the part-time water power developed at a flood-control dam might be absorbed efficiently. In such cases it takes a detailed and careful study to determine whether the power developed will pay the cost of plant installation. In many cases, the return would be less than the cost of the plant and in practically all cases at the present time the cost would be more than that of developing power by other methods. Future economic trends might change this balance and make development of plant under some such conditions economical.

The most important objection to the ordinary operation of a flood-control reservoir from the power standpoint is that of uncertainty of available power. The power plant must be capable of furnishing power on demand. It must have dependable capacity. Unless an installation can furnish power when required, other plant must be installed to meet the peak demands and the water-power plant loses a large part of its potential value. The flood-control reservoir operates only when there are floods (time of occurrence unknown); therefore, it cannot be depended on to reduce peak power plant installation. However, there are special cases where modified operation may correct this basic difficulty to some extent.

By sacrificing some measure of flood protection and power production, a combined use of the same storage is sometimes feasible. Floods can be ex-

pected to occur at certain seasons. Power demands for the locality have a certain seasonal variation. Operating schedules for plants in a system with other power producers might be arranged to permit the seasonal use of production from a water-power plant at the flood-control reservoir.

These factors may be combined to develop a rule curve for operating the reservoir to obtain the best possible use for both functions. Such a curve would consist of a schedule of elevations of desired water surface in the reservoir, set arbitrarily, and to be obtained throughout a year. For the conditions set forth in the preceding paragraph such a curve would provide that the reservoir be kept at a low elevation during the flood-producing period. As this period closes, increased elevations are permitted to provide increased head and storage for the low-water season. During the low-water season this storage is drawn down, regulating the power output and making certain that the reservoir is low at the beginning of the next high-water season. An interesting example of such operation is a stream in California that has both snow and rainfall run-off. A damaging flood only results from a combination of both snow and rainfall run-off. Therefore, a reservoir system may be designed to remain low until the snow run-off occurs and then either to store or to regulate the flow when it does occur, depending on whether or not it is combined with heavy rainfall. Enough storage is retained at the end of such a flood period to regulate the flow through the summer. This special case gives a fairly good operation particularly as the probable occurrence of storm rainfall is quite definitely limited to a rather short calendar period.

If storage capacity is available in excess of that required for flood-control purposes, combined interval storage may be considered (Class (*B-a*)). In effect, this usually is reduced to the task of determining the most economical power-plant installation and considering flood storage as a surcharge on the power reservoir. The flood surcharge on the power reservoir would consist of a certain zone of storage operated only in the interest of flood control. It might be operated to give complete protection for the maximum probable flood or to give protection from ordinary floods, depending on the economics involved. The operation of this zone would follow some one of the storage or retarding basin plans as discussed previously herein.

The power interval of this type of combined reservoir has a certain value in controlling or reducing floods that occur when normal power operation has the reservoir drawn down. The flood-control section also has some power value in that it furnishes additional flow and extra head during flood periods. Thus, a fair degree of mutual benefit is obtained, which adds to the combined value of the project. However, one important fact must always be considered in all combined projects; there must be some use for both functions. There must be a market for the power and there must be some damage hazard eliminated by providing flood protection. Until the value of both these uses exceeds the cost there can be no justification for construction even if the project may be physically feasible.

Flood Control and Irrigation.—In territory where rainfall is more or less deficient there is frequently agitation for a combination of irrigation and

flood protection by means of reservoirs (Class (*A*-*b*)). Such rainfall as occurs frequently causes floods. Storage of these flood waters to be released during later periods of drought would seem to offer considerable promise of combined use of the same storage space. If there was any regularity in the quantity or occurrence of the flood flow this principle would work very well.

However, floods vary so much in volume and time of occurrence that, in most cases, it is either impossible to obtain assurance of flood protection by having the reservoir empty at the time of flood occurrence or to carry the flood water over a protracted period of drought for irrigation. If land is to be prepared for irrigation, and if crops are to be grown, the water supply must be dependable. This would require the release of stored water in accordance with a rule based on a period of low supply. If a period of high supply occurs with a sharp concentration in the form of a flood—it is quite possible that the reservoir would be full, or nearly full, and that the flood would not be greatly diminished.

The second type of combined operation (Class (*B*-*b*)) (designation of separate storage bands within the same reservoir) offers more possibilities. There is a certain overlapping of the use of each band. Floods occur in certain seasons and a moderate draw-down of the irrigation storage may be permitted during these periods. On the other hand, when the flood period is over, a certain percentage of the flood-storage zone may be utilized for carrying over flood water for irrigation use.

Generally, in territory where irrigation is desirable, storage space is not particularly expensive. Thus, a large storage volume may be obtained for combined use. In some cases of existing reservoirs, flood protection is obtained as a more or less incidental benefit. At Lake Kemp, an irrigation and water supply reservoir, near Wichita Falls, Tex., it was found too expensive to re-locate several highways running through the site under existing conditions. The lake level, therefore, is kept 15 ft or more below the spillway crest. This surcharge has a volume equal to most floods for the tributary if a bank-full capacity is discharged through the conduits in the dam during the flood. Since the completion of the dam several floods have been equated by this means to down-stream channel capacity. The principle of a surcharge for extreme flood protection in the case of an irrigation reservoir is worthy of considerable study wherever there is a concentration of value with potential flood damage below sufficient to justify a considerable expenditure for flood protection.

Flood Control and Water Supply.—The combination of flood protection with domestic or industrial water supply (Class (*A*-*c*)) is very similar to the combination with irrigation, except that it applies to a much wider territory. The main difficulty in all cases is in the development of dependable supply for domestic or industrial use without, at the same time, filling up the volume required for the flood-control plan.

As with irrigation it might almost be stated as axiomatic that no combined project is possible unless there is more storage than is required for flood control and unless a combined operation of separate storage zones is possible

(Class (*B-c*)). In effect, this means a flood surcharge on a water supply reservoir. In many cases this is physically feasible, but the funds that can be raised to construct the flood-control surcharges are not sufficient to pay the additional cost. Hence, the industry builds only for itself, or the town builds for a water supply only sufficient for its needs in the near future. Many good dam sites are occupied by structures very much smaller than is feasible or desirable from the standpoint of efficient use of the water resources of the stream on which they are located; but to build the higher dam would be costly, and although flood benefits are large at the time a flood is occurring, they are seemingly non-existent when it comes to raising funds to provide structures for their realization. It is not the purpose of this statement to convey the impression that surcharges on water supply or on irrigation reservoirs are generally desirable. They usually would cost more than the capitalized flood damage that they would eliminate, but justified or not, they are almost never built because of the difficulty of collecting the additional money required from those who might be protected. They do offer about the only practicable combined operation in general and would give fairly good protection in many cases.

Flood Control and Navigation.—The combination of reservoir storage for open-river navigation with flood control (Class (*A-d*) or Class (*B-d*)) is like that of irrigation and water supply. It offers even less possibilities than either of these. This is due to the fact that a relatively large increase of low-water flow is needed ordinarily to provide modern open-river navigation. This alone requires a large storage. This means that insufficient storage is left for flood control and that unsatisfactory results are obtained for either use.

There is one combined navigation and flood-control plan, however, that can be applied in some cases, and it always warrants consideration. If canalization is being considered on a relatively large stream where the dam sites are such that considerable excess storage in the form of a flood surcharge can be obtained at each pool without causing excessive damage in the valley, this type of combination may be feasible. By routing the flood volume through the several pools of the system, the outflow at the lower end may be regulated to give a considerably lower flood crest. This method in effect merely creates a stream with a large amount of valley storage to equate flood peaks. Obviously, it is predicated on potential navigation being sufficient to justify the cost of the lock and dam system, with flood control contributing only that amount necessary to pay for the surcharge on the various navigation dams.

Combined Operation in Interest of Three or More Functions.—The combination of three or more of the uses stated involves most of the difficulties covered in the discussion for combining any two uses. Under certain conditions the combination of three uses is better than two. Generally, this results by reason of the same use benefiting two of the three functions. A reservoir with a flood-control surcharge on a navigation dam may also permit the development of water power. An irrigation dam may be used to develop power by depending to some extent on the flood-control surcharge to furnish

the extra water for power operation. Essentially, the engineering problems involved are similar to those outlined previously herein.

Summary.—In most combined projects the problem of distributing the cost and return is so complicated that it almost always requires some arbitrary allocation to the various functions, particularly if any Government agency is a party to the project. The practical difficulties encountered by an organization developed to manage an irrigation project that is combined with a water-power business have been well stated[5] by Charles C. Cragin, M. Am. Soc. C. E. This difficulty would apply to the operation of any combined project. Each function requires its special type of organization, and the development of a single organization to handle more than one function offers many difficulties. The most important consideration in arriving at a conclusion as to the justification for a combined project is that each function must have a value—a real value—that will justify contribution toward the cost of construction, mainenance, and operation of the project developed.

Reservoirs Combined with Other Structures

Flood-control projects may be developed by combining reservoirs with: (*A*) Side channels; (*B*) over-bank floodways; (*C*) channel improvements; (*D*) closure of outlets; and, (*E*) levees.

Although a physically possible combination, Combination (*A*) of reservoirs supplemented by side channels is seldom feasible economically. More or less duplicate structures are required to control the excess peak beyond the capacity of the main stream. Ordinarily, either the reservoir system or the side channel can be made large enough in itself at less cost than that of providing both systems of structures.

To combine reservoirs and over-bank floodways has the same objections as Combination (*A*). However, there may be conditions when it would be feasible. If the floodway can be reduced materially by controlling a part of the peak in reservoirs, this reduction may more than balance the reservoir cost. A variation of this might be a combination in which reservoirs and the natural stream could control all but a relatively small part of the infrequent high floods. This part might be permitted, on such rare occasions, to overflow through certain areas of low development in the overflow plain. This necessarily assumes the cost of such floodways to be relatively small. Under these circumstances, the combination is worthy of detailed analysis. If both the floodway and the reservoirs involve large expenditures, it may be expected that either one or the other, made large enough to control the entire excess flow, will be less expensive than any combination.

Natural or artificial obstructions frequently produce reaches of limited channel capacity in sections of overflow areas. In many cases these obstructions can be removed or can be compensated for at moderate cost to give much larger channel capacity (Combination (*C*)). This automatically reduces the extent of reservoir control necessary by permitting more flow through the reservoirs or a less area to be controlled by reservoirs above the danger zone.

[5] "Development of Hydro-Electric Power as an Aid to Irrigation," by Charles C. Cragin, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1240.

The Miami River[5] flood-control project illustrates this principle. The channel was improved through the reaches at Dayton, Ohio, to permit a controlled flow greater than normal. This increased the peak flow permissible below the reservoirs and reduced their size to such an extent as to make a feasible combination.

It is possible that channel improvements in the form of cut-offs may so increase the natural flow in a danger zone as to make a feasible combination with reservoirs. Cut-offs must be analyzed carefully, however, and exhaustive studies made as to their probable ultimate effect. In some cases their effects may be only temporary. This may or may not be a serious difficulty. In other cases, the improvement obtained may persist for many years.

On a small stream in Iowa a series of cut-offs was made to produce a straight channel with increased capacity. After fifteen years this stream shows a marked tendency to regain its sinuous course. However, in this time, the improved channel has controlled or reduced sufficient floods to pay for its cost. In other cases, cut-off shortening persists for long periods. Cut-offs were made on the Mississippi River, near the mouth of the Red River, in 1831 and 1848, respectively. The shortening effected by these cut-offs has been only slightly regained in about 100 years. A series of interesting experiments on cut-off effects[6] was conducted by the U. S. Waterways Experiment Station, Vicksburg, Miss., in 1933. A cut-off on the Mississippi River was opened by dredging at Diamond Point, Miss., about 12 miles below Vicksburg, in 1933.

On many rivers, outlets or low banks permit overflow into side basins long before the banks begin to overflow generally (Combination (*D*)). This is particularly true on alluvial rivers where the banks are higher than the back country. By closing the outlets the quantity to be carried in the main stream may be increased, but the stage at which overflow begins is also increased. By increasing the stage of general overflow, much less reservoir capacity is required than without the closure, and a feasible combination may result.

Combination (*E*), of reservoirs with levees, is quite similar to that of reservoirs and closure of outlets. The levees may merely close the low places in the bank and by a moderate expenditure so increase the channel capacity as to reduce materially the reservoir storage required. In some cases of streams partly protected by existing levee systems, moderate improvement of these systems in conjunction with reservoirs to take the high peaks may give a feasible combination. In general, levees and reservoirs offer possibilities when a moderate expenditure for levees will give a system controlling the long, moderate floods, or portions of floods, with reservoirs controlling the sharp flood crests that, with a relatively small volume, might cause failure of the levee system.

Summary.—The substitution of other structures in a reservoir plan is often feasible if moderate expenditure will eliminate a large percentage of reservoir storage. Expensive reservoirs on certain tributaries may be elim-

[6] *Paper I*, U. S. Waterways Experiment Station, Mississippi River Comm., Vicksburg, Miss.

inated altogether if the channel capacity of the main stream can be increased to provide for the maximum probable flow from the tributary. In other cases, where a levee or floodway plan can control all but a relatively small portion of the flood crest, it may be found that a reservoir system is the least costly method of controlling such a peak.

Conclusions

Several primary considerations control the feasibility of a reservoir plan for flood control. The first consideration might be termed physical feasibility, expressed by the question, are there reservoirs with sufficient capacity so located above the area to be protected as to control that portion of the flood waters that may cause damage? This includes the selection of physically feasible dam sites, reservoir basins, and their location at points below the collecting areas for run-off and above the area to be protected. Generally speaking, reservoir control is feasible, physically. There is a wide range in the cost of storage required, but there are few cases in which such storage is not physically available.

The second consideration might be termed "operating feasibility" as defined by the proposition: Given the storage required to control a flood crest is it within the realm of probability that the reservoir can be operated properly? A reservoir that has been placed above a town to be protected, and fails, may cause more damage than the flood which it was designed to prevent. Even if it does not fail as a structure, it may fail in its function of flood control if the method of operation is not workable as a flood develops. In this paper many methods of operation have been discussed. Each has its advantages and disadvantages and conditions under which it might operate satisfactorily. There are, of course, other methods of operation. The method to be used depends largely on the characteristics of the flood that may be expected. The mission of each system must be kept carefully in mind in each case and the incidental damages as well as benefits considered. If a tributary is being regulated to protect an area on a main stream, the extent to which this regulation may interfere with tributary protection must be balanced against the value of protection gained on the main stream.

It is obvious that any storage system has certain inherent objectionable features: It involves the manipulation of gates according to some rule of design; the designer may not be the operator; or the operator may be restricted by hysteria in the community as the flood develops. He may fail to pass as much flow through the dam as the design calls for, with the resulting probable overtopping of the reservoir if the designer's estimate of flood flow is reached. However, detailed studies of many conditions indicate that many areas now subject to flood damage might be protected by a reservoir system the operation of which appears feasible.

The last and perhaps most important consideration is economic feasibility. A system of reservoirs may be physically feasible and may operate perfectly; but its construction may be entirely unwarranted due to excessive cost or considerably greater cost for the same measure of protection than that obtained

by other methods of protection. To obtain a favorable balance for a reservoir plan requires: (*a*) A high concentration of value in the overflow area to be protected, such as a large industrial plant, a town, a city, or some other unit subject to relatively high flood damage; (*b*) low cost of storage; (*c*) a relatively large channel capacity below the reservoir capable of accommodating all ordinary flows; (*d*) a relatively sharp short flood to be controlled; and, (*e*) the reservoir ordinarily located fairly close to the area to be protected.

In comparison with levee plans of protection, the reservoir has the advantage in controlling sharp floods, but it is at a disadvantage in controlling long flat-crested floods. In protecting farm lands in an overflow plain, a reservoir system frequently offers a much better solution of the local drainage problem in that it keeps stream levels low and permits run-off of local rainfall, whereas the levee system retains high stream levels and may interfere seriously with local drainage. However, there are few cases in which reservoir protection of farm lands can be justified economically.

Summary.—The reservoir has many disadvantages as a flood-control structure. It is difficult to locate and operate; nevertheless, the advantages of protecting an area by reducing the flood crest that reaches it are such that a reservoir solution should be analyzed carefully before any flood-control plan is adopted.

Acknowledgments

The writer wishes to acknowledge the assistance of David W. Persons, Jun. M. Am. Soc. C. E., in preparing this paper. He also wishes to acknowledge the co-operation of District Engineers at Memphis, Tenn., and Vicksburg, Miss., and Brig.-Gen. H. B. Ferguson, U. S. Army, President of the Mississippi River Commission, in permitting the use of the files of the Commission as a source of data.

by other methods of protection. To obtain a favorable balance for a reservoir plan requires that: A high concentration of values in the localities protected, such as a large industrial plant, a town, a city, or some other well adapted to reservoir control; (b) large volumes; (c) few sites of economy; relatively large channel capacity below the reservoir capable of accommodating all ordinary flows; (d) a relatively short flood to be controlled; and (e) the reservoir ordinarily located fairly close to the area to be protected.

In comparison with levee plans of protection, the reservoir has the advantage in controlling sharp floods, but it is at a disadvantage in controlling long flat crested floods. In protecting farm lands in an overflow plain, a reservoir system frequently offers a more satisfactory solution of the local drainage problem in that it keeps streams within their banks and permits run-off of local rainfall, whereas the levee system retains high stages in rivers and may interfere seriously with local drainage. However, there are few cases in which reservoir protection of farm lands can be justified economically.

Summary.—The reservoir has many disadvantages as a flood control structure. It is difficult to locate and operate; nevertheless, the advantages of protecting an area by reducing the flood crest that reaches it are such that a reservoir solution should be examined carefully before any decision on a plan is reached.

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The writer wishes to acknowledge the assistance of David W. Hudson, Jun. M. Am. Soc. C.E., in preparing this paper. He also wishes to acknowledge the cooperation of District Engineers at Memphis, Tenn., and Vicksburg, Miss., and Brig. Gen. H. B. Ferguson, U. S. Army, President of the Mississippi River Commission, in permitting the use of the files of the Commission as a source of detail.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

STRESSES IN SPACE STRUCTURES

BY F. H. CONSTANT,[1] M. AM. SOC. C. E.

SYNOPSIS

The analysis of any but the simplest space structures is usually a tedious operation because of the three-dimensional nature of the problem. In the method herein presented most of these difficulties are avoided by substituting for the forces and stresses of the three-dimensional structure, a corresponding system of co-planar forces, the magnitudes of which may then be found by any of the well-known, simple processes ordinarily applied to the solution of planar structures. These co-planar forces, in turn, are readily translated back into the corresponding space stresses by dividing them by the cosine of the slope of the space member.

The method was originated by Professor Benjamin Mayor,[2] of the University of Lausanne, at Lausanne, Switzerland, but his presentation is based on the principles of the linear complex, a mathematical field little known to engineers and extremely difficult to follow. In this paper the writer has avoided the use of the complex and has based the theory on well-known principles of mechanics.

A system of forces is reducible at any point in space to a single resultant, R, and to a couple, S. Both may be represented by vectors acting at the point, making an angle, α, with each other. The vector, S, acts perpendicular to the plane of the couple, and by the usual convention, points in the direction of translation of a right-handed screw when rotated in the direction of the couple. For a clockwise couple, S will point toward the back of a watch and will be called positive.

If the point of reference is moved, R will remain invariable, but S, in general, changes in magnitude and slope. There is one line, the "central axis," parallel to R, in which the direction of S coincides with that of R; in other words, $\alpha = 0$ and both R and S_0 act along the central axis. (The particular value of S at the central axis is S_0.)

NOTE.—Discussion on this paper will be closed in September, 1934, *Proceedings*.

[1] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

[2] "Introduction à la Statique Graphique des Systèmes de l'Espace," par Benjamin Mayor, 1926.

At any other point, distant d from the central axis, R still remains parallel to that axis, but S lies in the plane normal to d and makes the angle, α, with R represented by the relation,

$$\tan \alpha = \frac{R\,d}{S_0} \quad \text{.........................(1)}$$

When d is infinite, $\alpha = 90$ degrees. Every value of S has the common property that its component in the direction, R, is constant and equal to S_0, or,

$$S \cos \alpha = S_0 \quad \text{.........................(2)}$$

In fact, every Vector S is tangent to a certain helix that is wrapped around a circular cylinder. The helix becomes steeper as the radius of the cylinder is less.

Besides the reduction to the screw type the system is also reducible to two resultants not lying generally in one plane. This reduction may be made in an infinite number of ways. The line of action of one of the vectors, F, may be given, and the other, F', may be required to lie in a plane having a given slope. The position of F' in this plane then becomes determinate and is said to be conjugate to F.

In Fig. 1, the line, $C\,A$, is the central axis which, for convenience, will always be assumed to be vertical. At some point, O, distant d from the line, $C\,A$, the system is equivalent to the vertical resultant, R, and a couple lying in some plane, $D\,E\,L\,G$, and represented by the normal vector, S. Assume, first, that one of the conjugate forces, $F = O\,J$, lies in the vertical plane containing $C\,A$, and that it intersects the latter at C. It is required to find its conjugate line in a horizontal plane. Through C draw a horizontal plane, $B\,C\,D\,E$. The line, $E\,D$, is the intersection of Planes $D\,E\,L\,G$ and $B\,C\,D\,E$. If Vector $O\,J$ is one of the conjugate forces its vertical component must be equal to R. Its horizontal component, $O\,H$, and an equal and opposite force, $E\,K$, acting along $E\,D$, will form a couple equal to S, because:

$$\overline{O\,H}\,l = \frac{R\,l}{\tan \phi} = \frac{R\,l\,d}{h} = \frac{S_0\,l \tan \alpha}{h} \text{ (from Equation (1))} = \frac{S\,l \cos \alpha \tan \alpha}{l \sin \alpha} = S$$

Therefore, $\overline{O\,J} = F$, and $\overline{E\,K} = F'$, are conjugate forces exactly equivalent to the original system.

Let b be the horizontal distance of F' from F_h. Then,

$$b = \frac{h}{\tan \alpha} = \frac{d \tan \phi}{\tan \alpha} \quad \text{.........................(3)}$$

From Equation (1), $\tan \alpha = \dfrac{R\,d}{S_0} = \dfrac{d}{a}\left(\text{when } a = \dfrac{S_0}{R}\right)$; whence,

$$b = a \tan \phi \quad \text{.........................(4)}$$

The horizontal position of F' (or its projection) depends then only on $\dfrac{S_0}{R} = a$, and the slope of F. As nothing is known about the original (central) system, $\dfrac{S_0}{R} = a$ may be given any convenient value suitable for the scale of the

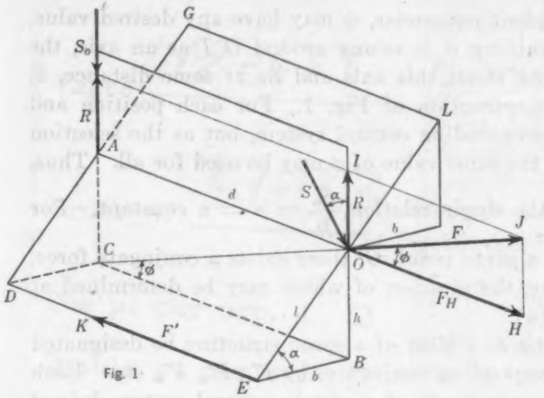

Fig. 1

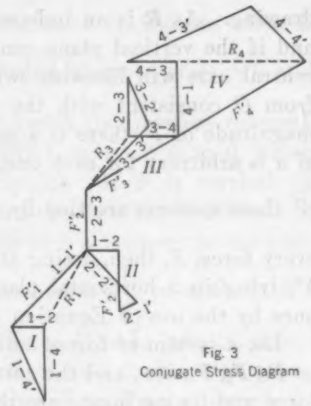

Fig. 3
Conjugate Stress Diagram

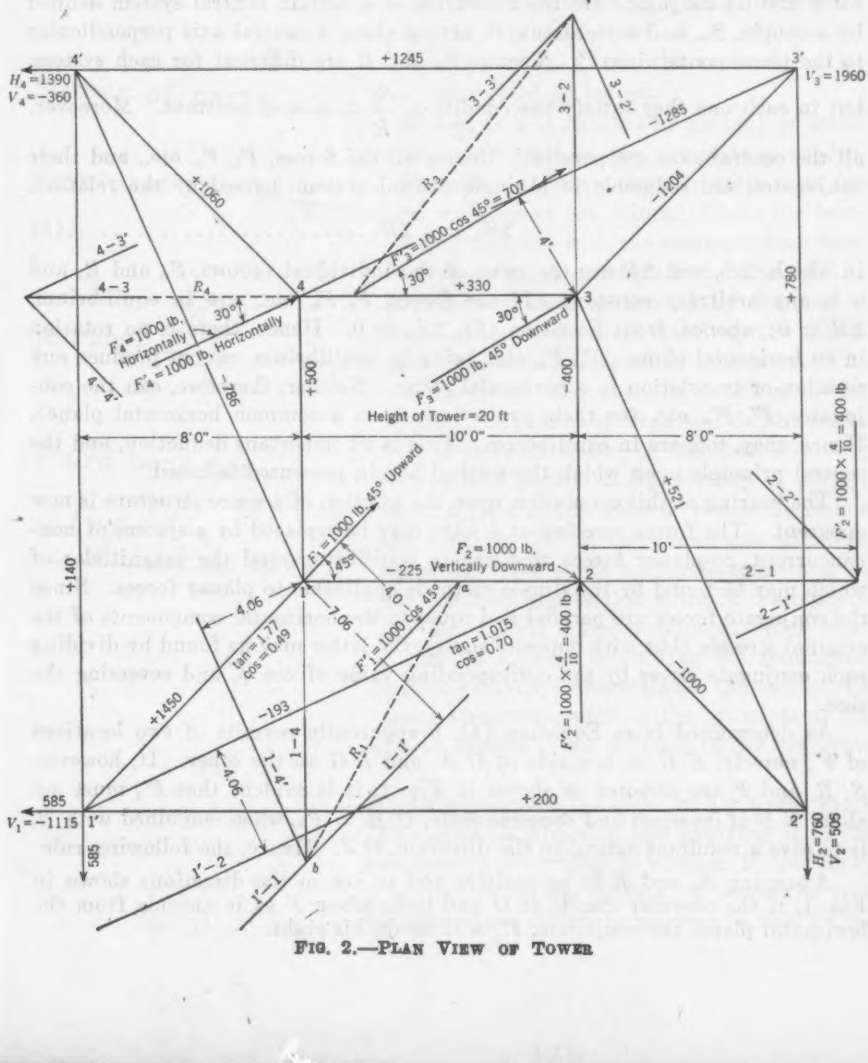

FIG. 2.—PLAN VIEW OF TOWER

drawing. As R is an independent parameter, ϕ may have any desired value, and if the vertical plane containing F is swung around $O\ I$ as an axis, the central axis will likewise swing about this axis and lie at some distance, d, from O consistent with the construction of Fig. 1. For each position and magnitude of F, there is a corresponding central system, but as the selection of a is arbitrary for each one, the same value of a may be used for all. Thus, all these systems are tied by the single relation, $\frac{S_0}{R} = a =$ a constant. For every force, F, then, acting at a given point, O, there exists a conjugate force, F', lying in a horizontal plane, the position of which may be determined at once by the use of Equation (4).

Let a system of forces acting at a joint of a space structure be designated by F_1, F_2, F_3, etc., and the corresponding conjugates by F'_1, F'_2, F'_3, etc. Each force and its conjugate are the resultants of a certain central system defined by a couple, S_0, and a resultant, R, acting along a central axis perpendicular to the plane containing F'. Vectors S_0 and R are different for each system, but in each one they satisfy the condition, $\frac{S_0}{R} = a =$ a constant. Moreover, all the central axes are parallel. Hence, all the forces, F_1, F_2, etc., and their conjugates, are reducible to a single central system defined by the relation,

$$\Sigma S_0 = a\ \Sigma R \qquad (5)$$

in which, ΣS_0 and ΣR are the sums of the individual vectors, S_0 and R, and a is any arbitrary constant. If the forces, F_1 F_2, etc., are in equilibrium, $\Sigma R = 0$; whence, from Equation (5), $\Sigma S_0 = 0$. Hence, there is no rotation in an horizontal plane. F_1, F_2, etc., being in equilibrium, cannot produce any rotation or translation in a horizontal plane. Neither, therefore, can the conjugates, F'_1 F'_2, etc. (or their projections upon a common horizontal plane). Hence, they, too, are in equilibrium. This is an important deduction, and the central principle upon which the method herein presented is based.

The bearing of this conclusion upon the solution of a space structure is now apparent. The forces meeting at a joint may be replaced by a system of non-concurrent, co-planar forces that are in equilibrium and the magnitudes of which may be found by the simple methods applicable to planar forces. Since the conjugate forces are parallel and equal to the horizontal components of the original stresses (but with opposite signs), the latter may be found by dividing each conjugate stress by the corresponding value of cos ϕ, and reversing the sign.

As determined from Equation (4), b apparently permits of two locations of F', namely, $E\ D$ on one side of $O\ A$, and $L\ G$ on the other. If, however, S, R, and F are directed as shown in Fig. 1, it is evident that F'_1 must act along $E\ D$ if its equal and opposite mate, $O\ H = F_h$, when combined with R, is to give a resultant acting in the direction, $O\ J$. Hence, the following rule:

Assuming S_0 and R to be positive and to act in the directions shown in Fig. 1, if the observer stands at O and looks along F as it ascends from the horizontal plane, the conjugate, F', will lie on his right.

This rule holds whether F acts away from or toward O, because when R is negative, S is likewise negative ($\frac{S_o}{R}$ always being assumed positive), and F' will still have the position shown in Fig. 1, but with its direction reversed.

If F is horizontal ($\phi = 0$), from Equation (5) $b = a \tan \phi = 0$, which indicates that the projections of F_h and F' coincide. If F is vertical ($\phi = 90°$), $F' = 0$, and b is ∞. Therefore, F' does not affect the conditions of translation of the conjugate forces; but it does produce a moment in the horizontal plane equal to S_o. In this case $R = F$, and from $\frac{S_o}{R} = a$, $S_o = R\,a = F\,a =$ moment of F' about the central axis. Since F' is at infinity, this is equivalent to saying that it enters into the equations as a couple with a constant moment of $S_o = F\,a$. Fig. 1 shows that when $F = R$ acts upward, the conjugate moment, $F\,a$, is clockwise. If F is nearly vertical, F' will fall beyond the limits of the drawing. In this case, F may be replaced by its vertical and horizontal components and treated as before.

The solution of the conjugate stresses is best effected by graphic or semigraphic methods. Only the horizontal projection of the structure is needed. Regarding each joint in turn as the center, O, draw the conjugate lines for all the external forces and members meeting at the joints. Those for horizontal forces pass through the joint and coincide with the corresponding lines in the plan view. The conjugates of the sloping members and forces will be parallel to their horizontal projections and may be located by the use of Equation (4). In a symmetrical dome or tower, there are relatively few different slopes, and most of the conjugate lines may be located by symmetry. The conjugate of a member will need to be drawn only once, as it will serve for the joints at both ends of that member. The steepest slope will fix the maximum practical value of a, unless it is more desirable, in the case of a very steep member, to work with its vertical and horizontal components.

When several members meeting at a joint of the space structure lie in one plane, their conjugates intersect in a point. If not more than one unknown force or stress lies without this plane, the value of its conjugate stress may be found by taking moments about the common intersection point of the remaining unknown conjugate stresses. When a joint is encountered at which more than two unknown stresses exist outside a plane containing the remainder, and the structure is apparently, but not actually, indeterminate, the well-known devices used in the analysis of space structures, such as the principle of the exchange of members, may likewise be applied to the solution of the conjugate stresses.

The conjugate stress, divided by the cosine of the slope, will give the space stress with its sense reversed. Changing its direction and applying it to the joint the stress is tension or compression according to whether it acts away from or toward the joint. To avoid this double reversal of sense the true direction of the horizontal components of the known forces may be used in the

conjugate system, and the resulting conjugate will then have the correct sense. The rule for vertical forces will then read: When Force F acts upward, the conjugate moment, $F\,a$, will be counterclockwise, and will lead to the correct signs of the actual stresses.

The following simple example will suffice to illustrate all the foregoing points. The symmetrical four-legged tower in Fig. 2 is 20 ft high, with loads at the four upper joints, as follows: 1 000 lb at Joint 1, directed 45° inward and 45° upward; 1 000 lb vertical and downward at Joint 2; 1 000 lb at Joint 3, directed outward at 30° with Member 3-4 and 45° downward; and 1 000 lb horizontal at Joint 4 at 30° with Member 4-3. A value of $a = 4$ ft was chosen for the location of the conjugates. For the diagonal member, 1-4′, $\tan\phi$ is 1.015, and since it slopes downward from the observer stationed at Joint 1, the conjugate lies on his left at a distance of $4 \times 1.015 = 4.06$ ft. Similarly, the conjugate for Member 1-1′ ($\tan\phi = 1.77$) lies 7.06 ft to his left as he looks down the column, 1-1′. Both are parallel to the respective horizontal projections. The conjugates for all the other members are quickly drawn from symmetry, and those for the external forces at Joints 1, 3, and 4, are located in a similar manner and are given the directions of the horizontal components of the actual forces. The force at Joint 2, being vertical and downward, is equivalent to a conjugate couple, $1\,000 \times 4 = 4\,000$ ft-lb, acting in a clockwise direction.

A graphical solution, somewhat akin to a Maxwell diagram (Fig. 3) has been adopted, although, of course, all the conjugate stresses could just as readily have been found by the use of the three equations of equilibrium. The graphical solution of Joint 1, for example, depends on the fact that the resultant of two of the conjugate forces must pass through the point of intersection and must balance the resultant of the other three. Thus, the conjugates, F'_1, of the member, 1-2, meet at a and their resultant, R_1, must pass through b, the point of intersection of the conjugates of the members, 1-4′, 1-1′, and 1.4. The force triangle (Fig. 3) corresponding to the first three gives R_1, and the stress in Member 1-2. In similar manner, the other ring stresses are found, including that in Member 1-4. Then, returning to the forces meeting at the point, b, with R_1 and the stress in Member 1-4 known, the remaining two conjugates are obtained by completing the force polygon (Fig. 3) for the joint. The other joints are solved in a similar manner. At Joint 2, in order to take account of the clockwise moment of 4 000 ft-lb, two equal and opposite forces are introduced, one at the joint and the other at the intersection point of the remaining three conjugates. Since these points are 10 ft apart, the magnitude of the forces constituting the clockwise couple must be 400 lb.

At the base of the tower, Joint 1′ is fixed, Joint 4′ has no horizontal restraints, and Joints 2′ and 3′ are restrained in directions perpendicular to Members 1′-2′ and 1′-4′, respectively. The vertical reactions are most quickly obtained by taking moments about the base joints of the known conjugate stresses of the column and diagonal meeting there. Thus, for V_1, taking

moments about Joint 1′:

$$\begin{array}{lcl}
\text{Moment of diagonal conjugate} = -707 \times 7.07 & = & -5\,000 \\
\text{Moment of column conjugate} = +135 \times 4.0 & = & +\ \ \ 540 \\
\hline
V_1 a \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & = & -4\,460 \\
V_1 = 4\,460 \div 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots & = & 1\,115
\end{array}$$

Since the moment, $V_1 a$, is counterclockwise V_1 acts upward.

The stresses in the bottom members and the horizontal reactions are obtained by a resolution of the conjugate stresses for the column and diagonal in the required directions, beginning with the free joint, 3′. The stresses and data for the solution of the structure are shown in the plan view (Fig. 2).

The advantages of the method increase with the complexity of the structure. If there are several cases of loading, the conjugates for the members remain unchanged, and only those for the external forces must be added in each case.

Conclusion

The use of articulated space structures has been limited in the United States, due possibly to the involved calculations or the more complicated character of the fabrication. There are many occasions, however, when they could be used to advantage. The method outlined herein should simplify the computation of the stresses considerably, and with the development of welding and other modern facilities in shop and field, such structures are likely to be used more frequently. The writer feels that a discussion of this aspect of the subject would be of interest to the profession.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

EXPERIMENTS WITH CONCRETE IN TORSION

By Paul Andersen[1], Assoc. M. Am. Soc. C. E.

Synopsis

Torsional stresses are developed in a structural member if it is subjected to the action of couples that lie in planes perpendicular to its axis.

In the ordinary building frame, provision seldom is made for taking care of stresses of this nature, and current literature on the design of reinforced concrete structures contains little on the subject. The attitude of the designing engineer is to avoid torsional stresses rather than to take them into account; and as a rule they can be avoided quite successfully.

There are, nevertheless, a number of types of structural members in which twisting forces occur and such forces should not be left to take care of themselves. The most important case is that of longitudinal balcony girders that support cantilever beams. Exterior floor-beams will develop torsional stresses due to the deflection of adjacent loaded panels. A skew arch submitted to vertical load and a rigid-frame portal sustaining a horizontal wind pressure also present problems in torsional resistance of reinforced concrete.

This paper deals with the general behavior in torsion of plain and reinforced concrete; its resistance against failure; and various types of reinforcement that will increase its ultimate torsional strength.

Notation

A summary of the notation introduced in this paper is presented herewith for the convenience of discussers:

a = one-half the side dimension of a square test prism;
b = spacing of bars measured on a horizontal cutting plane;
c = compressive stress in test cylinders; as a subscript, c, denotes "concrete";
e = a subscript denoting "edge";
h = an influence ordinate of shearing stress;
n = ratio of moduli of elasticities;
s = a subscript denoting "steel".

Note.—Discussion on this paper will be closed in September, 1934, *Proceedings*.

[1] Care, Portland Cement Assoc., Chicago, Ill.

t = tensile stress: t_c, permissible for concrete; t_{max}, maximum diagonal stress developed by a specimen without spirals;

v = shearing stress: v_e, along the edge of a cylinder produced by a twisting moment, M; v_ρ = shearing stress at a distance, ρ, from the axis of a cylinder; v_m = maximum shearing stress;

A = area: A_s, of steel bars;

E = modulus of elasticity: E', in shear;

F = a reinforcement efficiency factor that allows for surface tension taken by reinforcement;

K = a substitution factor; a unit stress;

M = moment; twisting moment;

N = number of bars, 45° to the axis, cut by a horizontal plane;

X = curve in Fig. 10;

Y = curve in Fig. 10;

θ = angle between two radii in a horizontal cutting plane;

ρ = radial distances in a horizontal cutting plane, measured from the axis of a cylinder; ρ_e, to the edge of a cylinder; ρ_s, to the steel reinforcement; ρ_c, to the point of excess permissible tension in the concrete;

ψ = angle of twist per unit length of cylinder.

Theoretical Considerations

Torsion is a state of pure shear. In the case of a circular cylinder, of a homogeneous and isotropic material,

$$v = E' \rho \psi \qquad (1)$$

in which, v is the shearing stress per unit area; E', the modulus of elasticity in shear; ρ, the distance from the axis of the cylinder; and ψ the angle of twist per unit length.

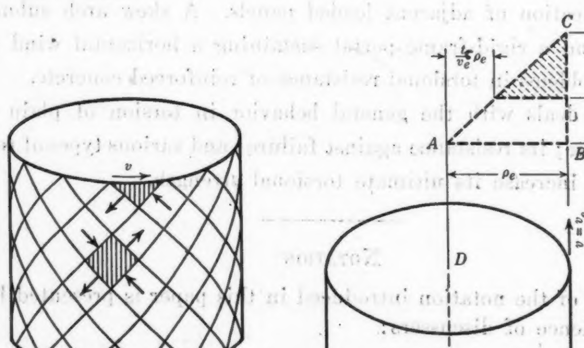

Fig. 1.

This state of pure shear is equivalent to tension in one direction and equal compression in the perpendicular direction. A rectangular element cut from the outer layer of a twisted shaft with its sides at 45° to the axis, therefore, will be subjected to the stresses shown in Fig 1(a), all of which are numeri-

cally equal to v. In the case of a material that is weaker in tension than in shear (such as concrete or cast iron) a crack along a helix inclined at 45° to the axis will occur when the ultimate tensile strength has been exceeded. It follows that the most effective reinforcement for a concrete member designed to resist failure in torsion is a spiral at an angle of 45° with the axis.

In the formulas that follow, the line of thought is analogous to that for the design of web reinforcement in a beam subjected to bending; namely, that sufficient reinforcement must be provided to resist tensile stresses in excess of the permissible tension for plain concrete. The line, *AC*, in Fig. 1(*b*), represents the variation in shearing stress along an arbitrary radius, *AB*. Let v_e be the shearing stress along the edge produced by a twisting moment, M, and let t_c denote the permissible tensile stress for plain concrete. The shaded triangle will then represent intensities of diagonal tension for which reinforcement must be provided.

Consider a small element at a distance, ρ, from the axis. The shearing force on this element in excess of the permissible stress is,

$$v = (v_\rho - t_c)\ \rho\ d\theta\ d\rho \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The tensile and compressive components of this force are numerically equal. Each one equals,

$$v = (v_\rho - t_c)\ \rho\ d\theta\ d\rho\ \cos 45° \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In accordance with the foregoing assumption, the sum of the moments with respect to the axis of the tensile components must equal the moment of the steel stresses with respect to the same axis, thus,

$$N\ t_s\ A_s\ \cos 45°\ \rho_s = \int_0^{2\pi}\int_{\rho_c}^{\rho_e} (v_\rho - t_c)\ \cos^2 45°\ \rho^2\ d\rho\ d\theta \ldots\ldots (4)$$

in which, N is the number of bars on an angle of 45° with the axis cut by a horizontal section; t_s, the allowable stress in steel bars; A_s, the cross-sectional area of a steel bar; ρ_s, the distance from Axis *AD* to the steel bars; v_ρ, the shearing stress at distance, ρ, from the axis; and $\rho_c = \dfrac{t_c\ \rho_e}{v_e}$ = a radial distance in Fig. 1(*b*).

In Equation (4) substitute:

$$v_\rho = \frac{\rho\ v_e}{\rho_e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and perform the integrations; thus:

$$N\ t_s\ A_s\ \rho_s = \frac{\pi\ \rho_e^3\ \sqrt{2}}{12\ v_e^2}\ (3\ v_e^4 - 4\ v_e^3\ t_c + t_c^4) \ldots\ldots\ldots (6)$$

or,

$$\frac{6\ \sqrt{2}\ \ N\ t_s\ A_s\ \rho_s}{\pi\ \rho_e^3}\ v_e^2 = (v_e - t_c)^2\ (3\ v_e^2 + 2\ v_e\ t_c + t_c^2) \ldots\ldots (7)$$

If K is substituted for $\frac{\sqrt{2}\; N\, t_s\, A_s\, \rho_s}{\pi\, \rho^3_e}$, Equation (7) may be written,

$$6\, v^3_e\, K = (v_e - t_c)^2\, (3\, v^2_e + 2\, v_e\, t_c + t^2_c) \quad \text{.............(8)}$$

If the concrete is assumed as not taking any part of the shearing stresses when these exceed the permissible tension for plain concrete, the case is as shown in Fig. 2. The formulas corresponding to Equations (4) and (7) are, respectively:

$$N\, t_s\, A_s \cos 45^\circ\, \rho_s = \cos^2 45^\circ \int_0^{2\pi} d\theta \int_{\rho_c}^{\rho_e} v_\rho\, \rho^2\, d\rho \quad \text{.........(9)}$$

and,

$$2\, K = v_e - \frac{t^2_c}{v_e} \quad \text{.........................(10)}$$

Equation (5) is identical for both cases.

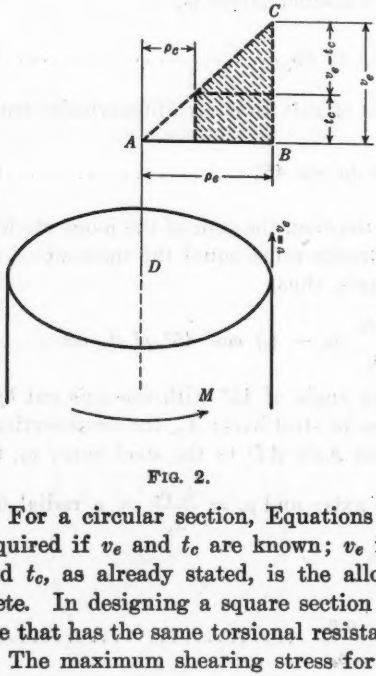

FIG. 2.

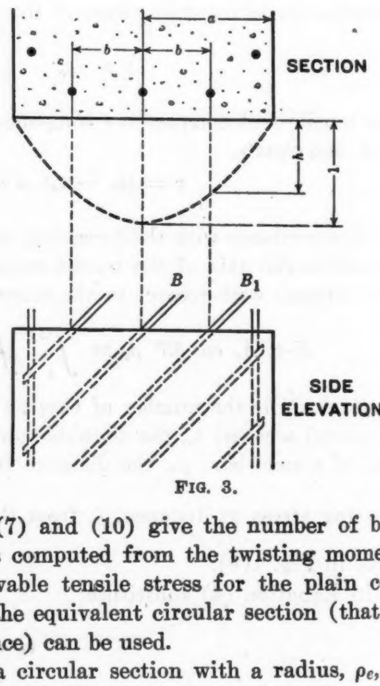

FIG. 3.

For a circular section, Equations (7) and (10) give the number of bars required if v_e and t_c are known; v_e is computed from the twisting moment, and t_c, as already stated, is the allowable tensile stress for the plain concrete. In designing a square section the equivalent circular section (that is, one that has the same torsional resistance) can be used.

The maximum shearing stress for a circular section with a radius, ρ_e, is,

$$v_m = \frac{2\, M}{\pi\, \rho^3_e} \quad \text{.........................(11)}$$

The maximum shearing stress for a square section with a length of side equal to 2 a is, approximately,

$$v_m = \frac{3\, M}{5\, a^3} \quad \text{.........................(12)}$$

Equating Formulas (11) and (12); $a^3 = 0.943\ \rho^3_e$ and $\rho_e = 1.02\ a$. Hence, the equivalent circular section is approximately the round section with a radius equal to one-half the side of the square section, or the inscribed circular section.

Fig. 3 shows a square section subjected to torsion. The shearing stress varies along the edge approximately as a second degree parabola; therefore, the reinforcement is stressed more highly at the center than near the corners. If, therefore, the equivalent circular section is considered to be reinforced with the same number of rods, it is clear that it would show greater resistance, due to the fact that its reinforcement would be equally stressed and, therefore, better utilized. Consequently, if Equations (7) and (10) are used for a square section to express the surplus tension to be taken by the reinforcement, a factor, F (which is less than one) must be introduced. Therefore,

$$6\ K\ F\ v^3_e = (v_e - t_c)^2\ (3\ v_e + 2\ v_e\ t_c + t^2_c) \qquad (13)$$

or,

$$2\ K\ F = v_e - \frac{t^4_c}{v^3_e} \qquad (14)$$

In Equations (13) and (14), F is a reinforcement efficiency coefficient relating to the variation in tensile stress along the edge. The efficiency coefficient for the square section shown in Fig. 3 is,

$$F = \frac{4 + 8\ h}{12} = \frac{4 + 8\left(1 - \frac{b^2}{a^2}\right)}{12} = 1 - \frac{2}{3} \times \frac{b^2}{a^2} \qquad (15)$$

Previous Investigations

Experiments with plain and reinforced concrete in torsion are recorded as early as 1904 at Stuttgart, Germany; in 1910 and 1911, by Professors Bach and O. Graf;[2] and, in 1920, by Professors Graf and E Mörsch, of the Engineering College of Stuttgart.[3] Further tests were made in 1922 by Messrs. Young, Sagar, and Hughes, of the University of Toronto, Toronto, Ont., Canada.[4]

Apparatus and Test Specimens

The testing, under torsion, of forty-eight specimens will be reported in this paper. The specimens were divided into six types as shown in Fig. 4, namely:

Type R: Six circular specimens of plain concrete; the control cylinder strength varied from 2 033 lb per sq in., to 5 535 lb per sq in.

Type B_1: Nine square specimens reinforced with $\frac{1}{2}$-in. square corner rods; the control cylinder strength varied from 2 110 to 5 240 lb per sq in.

[2] *Deutscher Ausschuss für Eisenbeton*, Heft 16, 1912.

[3] "Der Eisenbetonbau," von E. Mörsch, Sixth Edition, Vol. I, Pt. II, Stuttgart. 1929, pp. 323–335.

[4] *Bulletin No. 3*, Univ. of Toronto, Univ. of Toronto Press, 1922, pp. 145–169.

Type B_2: Nine square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round ties, spaced 6 in.; the control cylinder strength varied from 1 950 to 5 300 lb per sq in.

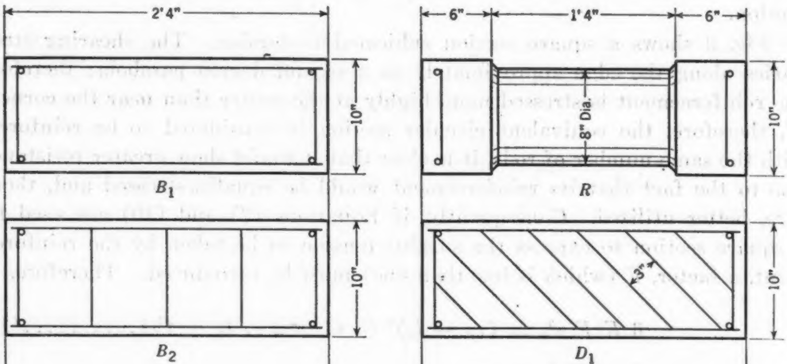

FIG. 4.

Type B_3: Nine square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round ties spaced 3 in.; the control cylinder strength varied from 2 090 to 5 190 lb per sq in.

Type D_1: Six square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round 45° spirals; the spacing of the spirals was 3 in., measured perpendicular to the 45° direction; the cylinder strength varied from 2 481 to 5 250 lb per sq in.

Type D_2: Nine square specimens reinforced with $\frac{1}{2}$-in. square corner rods and $\frac{5}{32}$-in. round 45° spirals; the spacing of the spirals was 2 in., measured normal to their direction; the cylinder strength varied from 1 950 to 5 300 lb per sq in.

The Portland cement used in manufacturing the concrete passed the standard specifications of the American Society for Testing Materials. Sieve analyses of the well-graded torpedo sand from the Wabash River, at Attica, Ind., showed a fineness modulus of 2.89 and a surface modulus of 18.5. The gravel, also from Attica, was graded from $\frac{1}{4}$ in. to 1 in. in size.

Concrete for all test specimens was machine-mixed for 4 min. The consistency is indicated by slumps of $4\frac{1}{2}$ to $7\frac{1}{2}$ in. for the various batches, with a mean value of 6 in., and an average flow (percentage increase in base diameter of flow cone) of 145. All specimens were cured moist for 28 days before testing.

The reinforcement consisted of hard-grade steel for corner rods and annealed black, soft-grade wire for spiraling and ties. It was subjected to the usual standard tensile tests, the results of which are given in Table 1.

As indicated in Fig. 4, the ends of the square specimens were not made larger than the test piece. However, in order to prevent local failures of the concrete at the ends, reinforcement in the form of $\frac{3}{4}$-in. square steel rods was embedded to furnish bearing against the structural steel grips of the testing machine. A standard torsion machine with a capacity of 230 000 in-lb was used.

TABLE 1.—Tensile Tests of Reinforcement

Size, in inches	Shape	Area, in square inches	Elastic limit, in pounds per square inch	Ultimate strength, in pounds per square inch	Percentage elongation in 8 in.
0.5	Square	0.25	78 800	134 800	7.5
0.1485	Round	0.01732	43 300	57 700	17.5

Fig. 5 shows the set-up for a square specimen. Strain-gauge readings in the 45° directions were taken on three sides of the test piece by a 2-in. Berry gauge. On the fourth side strains, also in the 45° directions, were recorded by two extensometers. Steel plugs for the extensometers as well as for the drilled strain-gauge holes were set and grouted in the concrete about two weeks before the day of testing. The angular distortion was measured as the difference in rotation between two sections 16 in. apart. To each section was attached a steel frame that made contact with the specimen in four points; the frame carried a horizontal steel bar. The angles of rotation of these bars were determined by a level-bar with attached screw micrometer.

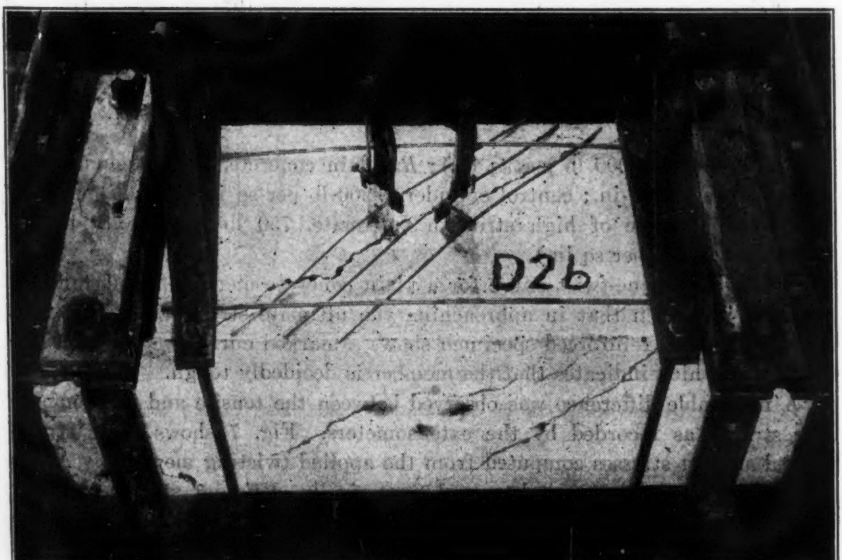

Fig. 5.—Set-Up for Square Specimen

The angle of twist was measured at intervals ranging from 5 000 to 15 000 in-lb; the interval was generally made smaller as the ultimate strength of the specimen was approached. Strain-gauge and extensometer readings were recorded at the same time as readings on the torque-indicating mechanism.

Concrete in Torsion

Fig. 6 shows the stress-detrusion diagrams for three typical types of specimen, namely, R_1, plain concrete, low strength (ultimate, 250 lb per sq in.;

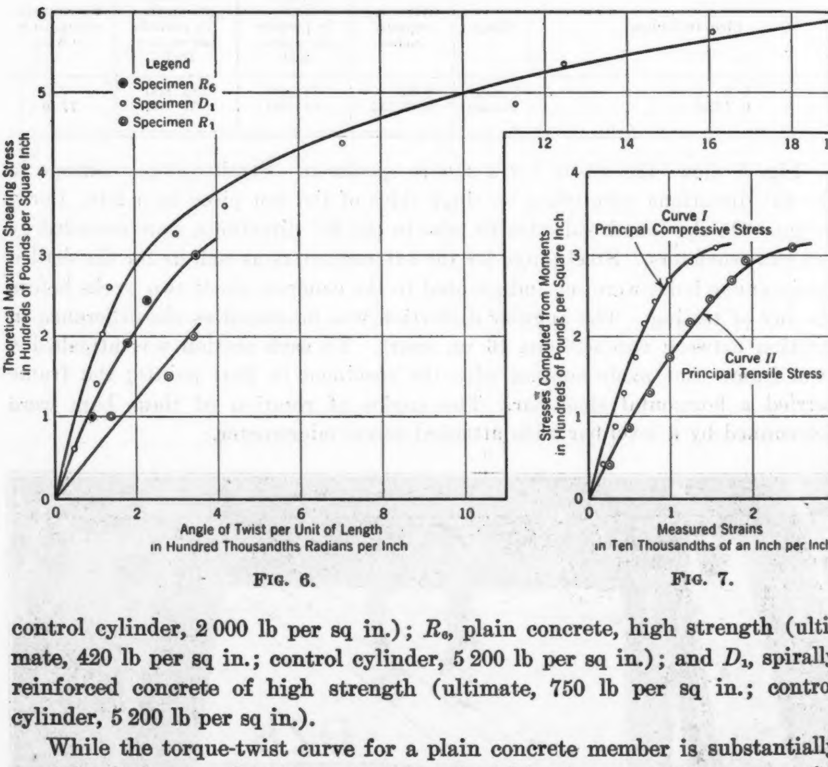

FIG. 6. FIG. 7.

control cylinder, 2 000 lb per sq in.); R_6, plain concrete, high strength (ultimate, 420 lb per sq in.; control cylinder, 5 200 lb per sq in.); and D_1, spirally reinforced concrete of high strength (ultimate, 750 lb per sq in.; control cylinder, 5 200 lb per sq in.).

While the torque-twist curve for a plain concrete member is substantially straight, it is seen that in approaching the ultimate stress, the torque-twist curve of a spiral reinforced specimen shows a marked curvature and considerable twist, which indicates that the member is decidedly tough.

A noticeable difference was observed between the tensile and the compressive strains as recorded by the extensometers. Fig. 7 shows these strains plotted against stresses computed from the applied twisting moments. As the two principal stresses must be equal numerically, different strains can only be accounted for by different moduli of elasticity, or by differences in the value of Poisson's ratio, in tension and compression. This difference was more pronounced for low-strength, than for high-strength, concrete.

Table 2 gives the modulus of elasticity in shear for plain concrete of varying torsional strengths. These are secant moduli determined at about three-fourths the ultimate strength.

TABLE 2.—MODULUS OF ELASTICITY IN SHEAR

Compressive strength of control cylinder, in pounds per square inch	Ultimate torsional strength, in pounds per square inch	Modulus of elasticity in shear, in pounds per square inch
2 000	250	1 420 000
2 250	300	1 380 000
3 300	315	1 440 000
Unreliable	350	1 540 000
5 200	420	2 140 000

The state of stress of a concrete member in torsion resembles that in a web of a concrete beam in bending; that is, the governing stress is diagonal tension. Under torsional shear a round shaft of concrete, invariably, will fracture along a helix the angle of which is 45° with the axis. The fracture occurs where the tensile stress is a maximum, and indicates that concrete has a lower resistance to tension than to shear. Fig. 8 shows a typical torsion fracture of a round specimen.

Although the state of stress for a square section is more complicated than that for a circular section, a similar distribution occurs, and the square specimen develops fractures at angles with the axis of approximately 45 degrees. Fig. 5 shows a characteristic torsion failure for a square test piece.

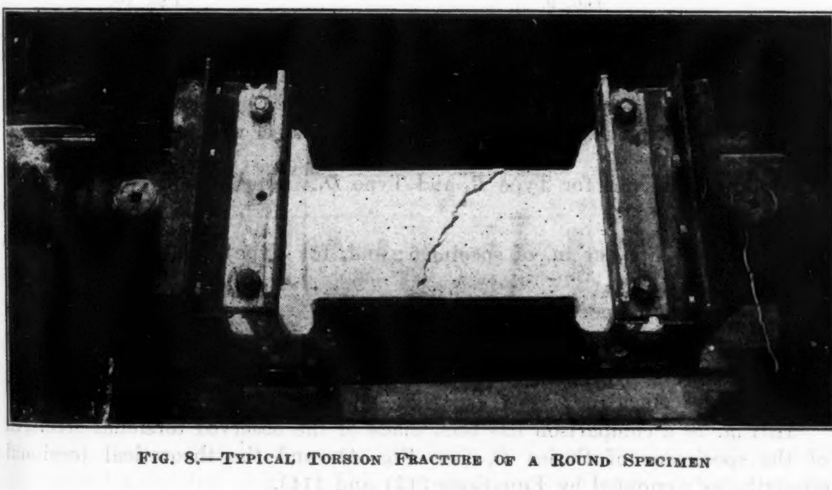

FIG. 8.—TYPICAL TORSION FRACTURE OF A ROUND SPECIMEN

ULTIMATE STRENGTH

The ultimate torsional moments were carefully recorded for each specimen. The corresponding maximum shearing stresses were computed by Equation (11) for a circular section with a radius, ρ_e; and by Equation (12) for a square section with a side equal to $2a$.

These stresses were plotted against the ultimate compressive strengths of the corresponding control cylinders in Fig. 9 and a straight line was fitted to each set by the method of least squares. Fig. 9 shows that the presence of

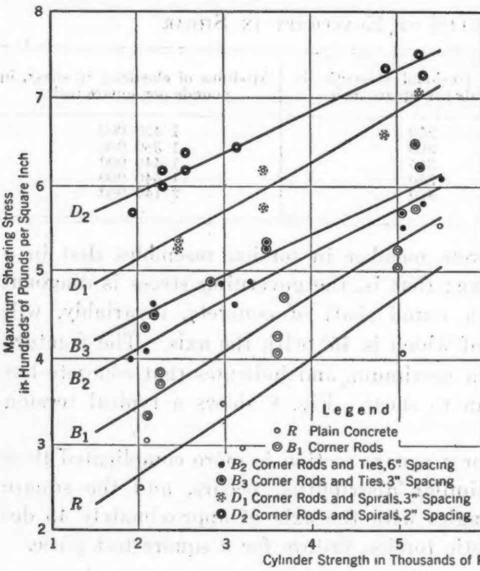

FIG. 9.

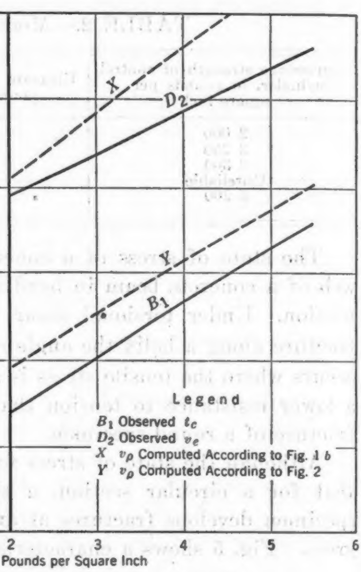

FIG. 10.

corner rods adds to the torsional strength; that the addition of ties will further raise the ultimate strength; and, also, that the most effective type of reinforcement used in these tests is the 45° spiral.

It is interesting to note that the quantity of steel wire per unit length of specimen is the same for Type B_3 and Type D_1. Thus, for Type B_3, $4 \times \frac{9.5}{3} = 12.7$ in. of wire per in. of specimen; and, for Type D_1, $4 \times \frac{9.5 \times \sqrt{2}}{3 \times \sqrt{2}} = 12.7$ in. of wire per in. of specimen.

By comparing the lines of Type B_3 and D_1, in Fig. 9, it is to be noted that the ultimate strength is considerably higher for the spiral reinforcement than for tie reinforcement.

In Fig. 10 a comparison has been made of the observed torsional strength of the specimens of Series D_2 (see Fig. 4), and the theoretical torsional strengths as computed by Equations (13) and (14).

Specimens B_1 were reinforced with four corner rods; Specimens D_2 had corner rods of the same size and additional close spirals. It seems natural to assume that the higher ultimate strength of Specimens D_2 is due entirely to the action of the spirals; or, in other words, that the differences between ordinates to Specimens D_2 and B_1 in Fig. 10 represent the quantities, $v_e - t_c$

(see Fig. 1 (*b*) and Fig. 2), in which, t_c is the maximum diagonal stress developed by a specimen without spirals, and v_e, the tensile stress sustained by a similar test piece having additional spiral reinforcement.

If it is desired to compute the ultimate strength, the quantity, $t_s\, A_s$, in Equations (13) and (14), equals the yield-point load in one wire and according to Table 1 is 750 lb.

The coefficient, F, can be calculated from Equation (15); thus,

$$F = 1 - \frac{2}{3} \times \frac{2.8^2}{5^2} = 0.791$$

Substituting these values, together with $\rho = 5$, $\rho_s = 4.5$, and $N = 9$, in Equations (13) and (14), gives,

$$519\ v^3_e = (v_e - t_c)^2\ (3\ v^2_e + 2\ v_e\ t_c + t^2_c) \ldots\ldots\ldots\ldots (16)$$

or,

$$173 = v_e - \frac{t^4_c}{v^3_e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which, Equation (16) represents the case illustrated in Fig. 1(*b*) and Equation (17), the case indicated in Fig. 2.

If, in Equations (16) and (17), values of t_c are inserted corresponding to different cylinder strengths, c, as indicated by Line B_1 in Fig. 10, equations with one unknown, v_e, will result. These equations can be solved by trial. Corresponding values of t_c and v_e are given in Table 3.

TABLE 3.—VALUES OF t_c AND v_e CORRESPONDING TO DIFFERENT CYLINDER STRENGTHS

Values of c, in pounds per square inch	According to Fig. 1 (*b*)		According to Fig. 2	
	Tensile strength, t_c, for plain concrete	Shearing stress, v_e, along the edge of cylinder	Tensile strength, t_c, for plain concrete	Shearing stress, v_e, along the edge of cylinder
2 000	343	610	343	396
2 500	374	647	374	426
3 000	405	684	405	456
3 500	435	721	435	486
4 000	466	758	466	516
4 500	496	795	496	546
5 000	527	831	527	576

The lines connecting these points represent the theoretical torsional strengths of plain homogeneous members equivalent to specimens of Series D_3. As indicated in Fig. 10, the agreement between the observed v_e, and the value computed according to Fig. 1 (*b*), appears to be considerably better than that calculated according to Fig. 2.

CONCLUSIONS

Except for the German and Canadian experiments, already cited, few data have been published in regard to plain and reinforced concrete under torsion.

The conclusions that can be drawn from the experiments reported in this paper, and that more or less confirm the test results of these experiments, are:

1.—The modulus of elasticity in shear of concrete depends on its ultimate strength. High-strength concrete has a higher shearing modulus than low-strength concrete.

2.—Concrete in torsion fails in tension due to its low tensile strength. The safe working stress for concrete in torsion must be considerably less than its ultimate tensile strength; the factor of safety should not be less than 2.

3.—The addition of corner rods will increase torsional resistance. The writer believes, however, that this extra strength becomes proportionately less for larger sections; it should not be relied upon, therefore, in designing concrete members subjected to torsion.

4.—The addition of vertical web reinforcement will further increase the torsional resistance and can be considered in the design, provided this reinforcement forms completely closed ties such as are used in columns.

5.—The most effective type of reinforcement is the 45° spiral. It raises the ultimate torsional strength and adds considerably to the toughness of the member.

The deduction to be made from the experimental data presented in this paper is that the spiral reinforcement can be assumed to take practically all tensile stresses in excess of the ultimate tensile strength of the unreinforced concrete. If a suitable factor of safety is chosen, design formulas can be developed for structural concrete members on this basis (see Fig. 1(*b*)).

ACKNOWLEDGMENT

The experiments cited in this paper were performed in 1932 and 1933 in the Materials Testing Laboratory of the University of Illinois, Urbana, Ill., by Rex L. Brown, Assoc. M. Am. Soc. C. E., and the writer, under the direction of F. E. Richart, M. Am. Soc. C. E.

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PAPERS

WAVE PRESSURES ON SEA-WALLS AND BREAKWATERS

By David A. Molitor[1], M. Am. Soc. C. E.

Synopsis

A method of evaluating wave pressures that may be expected in a given locality is presented in this paper and the writer applies the results of his observations to the investigation of structural stability, or design, of sea-walls and breakwaters. Since all factors contributing to the solution of the problems herein considered are only approximately knowable, no theoretical approach was deemed advisable. However, all available observational data were used in evolving a method of analysis.

Engineering structures should be designed with a certain margin of safety, yet not wastefully. In the past many structures, as first designed and built, required rebuilding after each severe storm until a satisfactory structure was finally developed. The design method herein presented should enable one to appraise his problem within reasonable limits, and arrive at a fairly economic solution, according well with experience on existing structures of enduring stability. The concluding remarks and two illustrative problems substantiate this claim.

Certain data on wind velocities are included because of the labor involved in collecting this information which is not readily obtainable from the United States Weather Bureau records. Data on weights of cribs and on coefficients of friction, so essential and important in design and stability investigations, are given because the elements of judgment and experience enter largely into their choice. The meager textbook information available may not even be applicable, or it may be misleading.

Nature of the Problem

Sea-walls and breakwaters are exposed to the action of waves generated by high winds, sweeping over a considerable expanse of open water. The size

Note.—Discussion on this paper will be closed in September, 1934, *Proceedings*.

[1] U. S. Constr. Engr. for Superv. Archt., Treasury Dept., Lansing, Mich. (Formerly Prin. Designing Engr., Toronto Harbor Commrs., Toronto, Ont., Canada).

of a wave for any particular locality depends on the velocity of the wind, duration of the storm, depth of water, and the greatest distance over which the wind can act, provided the water is of sufficient depth for wave formation.

Assuming a constant velocity and direction of wind, the height of a wave will increase from zero at the windward shore to some limiting value which the assumed wind velocity will just maintain. This distance from the windward shore to the wave is called the "fetch" of the wind. When the fetch is sufficiently great and the depth of water is ample, the wave will attain the limit of height before reaching the leeward shore, continuing with a constant height until some resistance is encountered in shallower water or against a structure of some kind.

As a wave increases in size according to the distance out from the windward shore, its length also increases and so does its velocity of propagation. When the balance is finally attained between the wind velocity and the wave dimensions, the velocity of the wave no longer increases, but may continue without reduction for long distances even after the wind has subsided.

Assuming for the present that the height of a wave may be estimated for a given fetch and wind velocity, as established for a given location where it is proposed to construct a sea-wall or a breakwater, the next step is to evaluate the probable mass and velocity of such a deep-water wave and, finally, to trace its retardation and diminished dimensions as it advances into shallower water previous to striking the wall where its moving mass is stopped. In case the wall is situated so that the maximum wave cannot strike normally, there will be further reduction in resolving the wave force normally to the face of the wall.

Having finally decided on the probable size of wave that may be expected to arrive in front of a wall, the next step is to estimate the height to which this wave will be piled up when completely obstructed, and then to decide on the height to which the wall is to be built. If the wall is not constructed to the full height of the completely obstructed wave, a portion of the wave will go over the top and the wave will be only partly obstructed.

When a wave is partly or completely stopped by a wall, a portion, or all, of its energy is converted into a dynamic force, which in turn represents the static equivalent of the expended energy. The wall structure must be capable of resisting this static equivalent without exceeding certain requirements of structural stability and safety.

The foregoing phases of the problem will next be examined in detail, to show that each element may be quantitatively evaluated in a logical sequence. It should be remembered, however, that all formulas used in this connection are based on observations which are in themselves quite approximate, while a theoretical basis is only remotely possible. The observational data herein cited were mostly those reported in 1904 by the late Capt. (afterward Lt.-Col.) D. D. Gaillard, Corps of Engineers, U. S. Army,[2] supplemented by observations made by the writer in 1915, in connection with the harbor development, at Toronto, Ont., Canada.

[2] "Wave Action in Relation to Engineering Structures," by Capt. D. D. Gaillard, Corps of Engrs., U. S. A., *Professional Papers No. 31*, U. S. Corps of Engrs., 1904.

Wave Height, Wind Velocity, and Fetch

For a given wind velocity, V, in miles per hour, and a fetch, D, in statute miles, the height, h, in feet, may be estimated for any wave with the aid of the following formulas:

For values of D greater than 20 miles:

$$h = 0.17 \sqrt{V D} \qquad (1)$$

and, for values of D less than 20 miles:

$$h = 0.17 \sqrt{V D} + 2.5 - \sqrt[4]{D} \qquad (2)$$

which are based on the formulas of Thomas Stevenson,[3] after introducing wind velocity as a variable, and using statute miles instead of nautical miles. These formulas apply only to inland lakes where the fetch is not likely to exceed the distance subjected to a violent wind in a single direction.

With a few exceptions ocean waves rarely exceed 45 ft in height, corresponding to a wind velocity of 75 miles per hr and a fetch of about 900 miles, which represents an unusual combination of circumstances. Ocean storms are generally more or less local, and do not cover more than 50 to 100 miles. Since the oceans are of much greater extent than any possible storm, the fetch is restricted to the storm area, which may be anything from a few miles to, say, 900 miles, and no formula can be made to apply to such indeterminate conditions. Hence, the only reliable data relative to the height of ocean waves must be collected by direct observations for any given locality. An extensive collection of such observations was published by Captain Gaillard.[4]

Length of Wave in Terms of Height and Wind Velocity

The highest wave has the minimum length ratio, and as the wind subsides the height diminishes and the length ratio increases. Fig. 1 gives certain dimensions for waves which will be used throughout this paper. The period, t, of a wave, is the time, in seconds, for the crest to travel the dis-

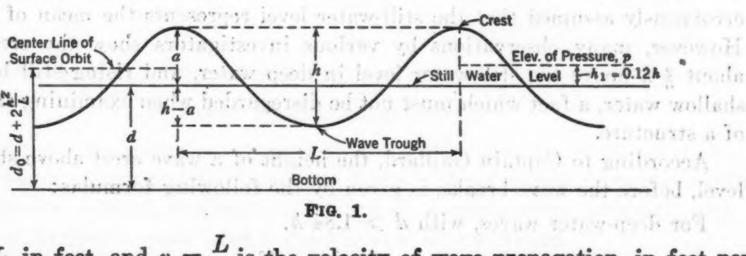

FIG. 1.

tance, L, in feet, and $v = \frac{L}{t}$ is the velocity of wave propagation, in feet per second.

(a) The Ratio, $\frac{L}{h}$.—The length ratio, $\frac{L}{h}$, varies between rather wide limits

[3] "Design and Construction of Harbours: A Treatise on Maritime Engineering," by Thomas Stevenson, Edition 2, Edinburgh, 1874.

[4] *Professional Papers No. 31,* U. S. Corps of Engrs., Table IX, p. 76.

depending on the velocity of the wind, V, the duration of a storm, and the depth of water in which the wave is formed. According to observations by Captain Gaillard in fresh and relatively shallow water, it varies between the limits, 9.1 and 15. A formula proposed herein, expresses this ratio in terms of wind velocity, V, as follows:

$$\frac{L}{h} = \frac{840}{V} \qquad (3)$$

Equation (3) gives the following values for wind velocities:

V	$\frac{L}{h}$	V	$\frac{L}{h}$
30	28.0	55	15.3
35	24.7	60	14.0
40	21.0	65	12.9
45	18.7	70	12.0
50	16.8	75	11.2

For ocean waves, Dr. Gerhard Schott[5] gives the values listed in Table 1.

TABLE 1.—VALUES OF $\frac{L}{h}$ FOR OCEAN WAVES

Description	Beaufort scale	Wind velocity, in miles per hour	Wave length, L
Moderate wind	5	28	33 h
Strong wind	6 to 7	35	20 h
Storm	9	56	17 h

The ocean waves are thus seen to be relatively longer than waves on the Great Lakes, as might be expected.

(b) Height of Wave Above Still-Water Level.—Since the crest of a wave is above the still-water level and the trough is below that level, it is sometimes erroneously assumed that the still-water level represents the mean of the two. However, many observations by various investigators show the crest to be about $\frac{2}{3}$ h above the still-water level in deep water, and rising still higher in shallow water, a fact which must not be disregarded when examining the safety of a structure.

According to Captain Gaillard, the height of a wave crest above still-water level, before the wave breaks, is given by the following formulas:

For deep-water waves, with $d > 1.84\ h$,

$$a = \frac{h}{2} + \frac{h^2}{L} \qquad (4a)$$

and for shallow-water waves, with $d < 1.84\ h$,

$$a = \frac{h}{2} + \frac{2\,h^2}{L} \qquad (4b)$$

[5] *Professional Papers No. 31*, U. S. Corps of Engrs., p. 87.

For deep-water ocean waves, Rankine[6] gives the formula:

$$a = \frac{h}{2} + 0.785 \frac{h^2}{L} \quad \text{(4c)}$$

The term, still-water level, as used herein, represents a line (see Fig. 1) for which the sectional area of the wave ridge is equal to that of the wave hollow.

(c) Depth of Water in Which a Wave Breaks.—While waves may break in deep water due to the action of increasing wind and other causes, they always break when they reach water of insufficient depth. A knowledge of this minimum depth is necessary in determining the maximum wave that may be expected to arrive at a certain structure located in shallow water. Undoubtedly, the roughness of the bottom has much to do with this subject and it would seem that the depth in which a wave breaks is much less for a rough bottom than for a smooth sandy bottom.

For fifty-five observations at the Duluth Canal by Captain Gaillard, for waves from 7 to 13 ft in height and a sandy bottom sloping about 1:40, the waves broke in a depth equal to 1.72 h. Similar observations on Lake Superior near Presque Isle Point, Mich., and Grand Marais, Mich., with a rough bottom and waves from 6 to 9 ft high, the waves broke in a depth equal to 1.3 h. For ocean waves at St. Augustine, Fla., with a strong wind blowing in the direction of wave travel, the waves broke in a depth of 1.25 h.

Therefore, if d_1 = the depth in which a wave breaks, the relations listed in Table 2 may be accepted for storm waves with the wind in the direction of the wave travel.

TABLE 2.—DEPTHS OF WATER, d_1, IN WHICH A WAVE BREAKS

Wind velocity, V, in miles per hour	Depth of water, d_1, in which a wave breaks (in terms of h, the height of wave)	Body of water	Bottom condition
Gentle	1.34 h	Lake Superior	Sandy
26	1.42 h	Lake Superior	Sandy
30	1.30 h	Lake Superior	Rough
40	1.84 h	Lake Superior	Sandy
40	1.25 h	Ocean	Sandy

DYNAMIC PROPERTIES OF WAVES

To this point the paper has dealt with wave dimensions, and the relations existing between these and the actuating force. The dynamic properties, including the velocity of propagation, energy of a wave, wave pressure, and height of a completely obstructed wave will now be considered.

(a) Velocity of Propagation.—According to Rankine,[7] for long waves or for waves in water that is very shallow compared with the wave length, the velocity of propagation, v, is given by the formula:

$$v = \sqrt{g\,(d + \tfrac{4}{3}\,h)} = 5.68\,\sqrt{d + \tfrac{4}{3}\,h} \quad \text{(5)}$$

[6] *Professional Papers No. 31*, U. S. Corps of Engrs., p. 37.
[7] *Loc. cit.*, p. 107.

in which, g = the acceleration due to gravity. According to Captain Gaillard, Equation (5) almost invariably gives results in excess of observed velocities. The theoretical velocity for deep-water waves with $d > \frac{L}{2}$, is:

$$v = \frac{L}{t} = \sqrt{\frac{g\,L}{2\,\pi}} = 2.26\ \sqrt{L} \quad\text{.................... (6)}$$

For shallow-water waves with $d < \frac{L}{2}$, Equation (6) becomes:

$$v = 2.26\,c\,\sqrt{L}\ \text{when}\ \frac{d_0}{L} = \frac{d}{L} + 2\left(\frac{h}{L}\right)^2 \quad\text{............ (7)}$$

in which, c, a constant expressing the ratio of the semi-minor to the semi-major axes of the surface orbits, has the values indicated in Table 3.

TABLE 3.—VALUES OF THE RATIOS OF AXES OF ORBITS

Depth ratio, $\frac{d_0}{L}$	Ratio, c, of semi-minor to semi-major axes of surface orbits	Ratio, β, of major axes to minor axes of surface orbits	Values of $\mu = c\,\beta$	Depth ratio, $\frac{d_0}{L}$	Ratio, c, of semi-minor to semi-major axes of surface orbits	Ratio, β of major axes to minor axes of surface orbits	Values of $\mu = c\,\beta$
0.05.....	0.552	3.286	1.814	0.30	0.977	1.047	1.023
0.10.....	0.746	1.796	1.340	0.35	0.988	1.025	1.013
0.15.....	0.858	1.358	1.165	0.40	0.994	1.013	1.007
0.20.....	0.922	1.177	1.085	0.45	0.997	1.007	1.004
0.25.....	0.958	1.090	1.044	0.50	0.998	1.004	1.002

The wave length, L, and velocity, v, are materially altered when the depth gradually decreases toward a shoal or shore, and Captain Gaillard proposed an empirical formula based on a number of observations made at the Duluth Canal, where the waves entered the canal at a depth of 26 ft, while the portion of wave outside the canal entered shallow water, gradually shoaling to 3.3 ft. The formula is:

$$v_2 = 0.9\ v\ \sqrt[4]{\frac{d_2}{d}} \quad\text{........................ (8)}$$

with a reduced height,

$$h_2 = h\ \sqrt{\frac{d_2}{d}} \quad\text{.......................... (9)}$$

in which, v is the velocity of the wave approaching from a depth, d, which should not be greater than $\frac{L}{2}$; and v_2 is the reduced velocity when the depth has shoaled to the lesser depth, d_2. A wave thus retarded will then have a reduced height, h_2.

In applying Equations (8) and (9) it is well to choose a depth of $\frac{L}{5}$ instead of $\frac{L}{2}$, because the dimensions of a deep-water wave are not appreciably altered until after the shoaling has become effective.

(b) The Energy of a Wave.—The theoretical energy, E, in foot-pounds, of a deep-water wave of length, L ft; height, h ft; and breadth of 1 ft, for fresh water, is given by:

$$E = 7.8\ L\ h^2 \left[1 - 4.935 \left(\frac{h}{L}\right)^2\right] \qquad (10)$$

For salt water, the coeffecient, 7.8, becomes 8.0.

For shallow-water waves the energy decreases from 2 to 10% below the value given by Equation (10) and the appropriate formula becomes rather complicated. As no particular use is made of this formula, the matter will not be further investigated.

(c) Wave-Pressure Formulas.—Having enumerated the various steps in arriving at the probable dimensions of storm waves for a given locality, the important question confronting the designing engineer, is to appraise with some reasonable degree of accuracy, the force that such a wave develops in striking a proposed structure. The most satisfactory manner of attacking this problem is by direct measurements of the force expended by impinging waves on a solid structure carrying a series of self-registering dynamometers.

Dynamometer measurements were carried on for many years by Thomas Stevenson, at Skerryvore Rocks, in the Atlantic Ocean, in 1843 and 1844, and at Dunbar Harbor, in 1858.[8] Several attempts to measure wave pressures on the Great Lakes were made at various times, but without gaining much information of value. The experiments made, in 1901 to 1903, by Captain Gaillard on Lake Superior, stand as the most valuable contribution to this subject.

Before mentioning any experimental results, the following conclusions, arrived at by Captain Gaillard as a result of his work are, that: (1) The impact of a wave does not resemble that of a solid body; (2) the pressure indicated by the types of dynamometers heretofore used are due to dynamic action only; (3) the pressures mentioned in Conclusion (2) apparently conform to the hydrodynamic laws governing the action of a current flowing normally against a submerged plane; (4) a mass of water in air projected with a certain velocity against a plane surface can produce no greater pressure than would be caused by the steady flow against this surface of a jet of equal cross-section having the same velocity and striking at the same angle; (5) from Conclusions (1) to (4), it follows by inference, that a mass of water projected against a submerged plane surface of considerably smaller area than the cross-section of the mass, can produce no greater pressure than would be caused by the steady flow at the same velocity of a current against a submerged plane surface of equal area and similar to the first; and (6) as the most destructive waves act for an appreciable period, the pressures they exert can properly be measured by suitably constructed dynamometers.

[8] *Professional Papers No. 31*, U. S. Corps of Engrs., p. 145.

The formula for hydrodynamic pressure on a submerged plate (in pounds per square foot) according to Dubuat,[9] is:

$$p = k w \frac{v^2}{2g} \quad \text{.........................} (11)$$

in which, v is the velocity of forward motion of the plate. The total velocity of the striking wave consists of the combined velocity of propagation, v, and the maximum orbital velocity, v_0, of a wave particle, so that Equation (11) should be modified to represent the maximum value (in pounds per square foot),

$$p_{\text{max.}} = \frac{k w}{2g} (v + v_0)^2 \quad \text{.....................} (12)$$

in which, k is an empiric coefficient evaluated from Captain Gaillard's observations for the Great Lakes as 1.30 to 1.71 for winds from 30 to 70 miles per hr. For ocean storm waves, k may be taken as 1.8. For fresh-water waves, $w = 62.4$ lb per cu ft and for salt-water waves, $w = 64.4$ lb per cu ft, while the acceleration due to gravity is $g = 32.2$ sec-ft.

The velocity of propagation for waves formed in any depth of water is given by Equation (7). The maximum orbital velocity of a wave particle, similarly, is:

$$v_0 = m\, c \sqrt{\frac{2 \pi g}{L}} \quad \text{.....................} (13)$$

in which, $m = \frac{\beta h}{2}$ = the semi-major axis of the elliptical orbit, and β is the ratio of the axes of the elliptical orbits. The numerical values of c and β, given in Table 3, render these expressions applicable to the solution of problems.

Introducing the numerical values of π and g, Equation (13) becomes,

$$v_0 = \frac{c \beta h \times 14.22}{2 \sqrt{L}} = 7.11\, \mu \frac{h}{\sqrt{L}} \quad \text{..............} (14)$$

and, with values of k, w, and g, substituted into Equation (12), the maximum unit wave pressure, in pounds per square foot, becomes:

$$p = \frac{k w}{2 g} (v + v_0)^2 \quad \text{.........................} (15)$$

in which, $v + v_0$ is the sum of Equations (7) and (13), or:

$$v + v_0 = 2.26\, c \sqrt{L} + 7.11\, \mu \frac{h}{L} \quad \text{..............} (16)$$

and $\frac{k w}{2 g}$ = 1.71 and 1.80, for fresh-water and for salt-water storm waves, respectively.

The values given for k are maximum for a 75-mile wind and may become as low as 1.3 for a 30-mile wind. When $\frac{d_0}{L}$ is equal to, or less than, 0.5, $c = \mu = 1.00$, which is the condition for deep-water waves.

[9] *Professional Papers No. 31*, U. S. Corps of Engrs., p. 174.

Having determined the maximum pressure exerted by a striking wave, it is necessary to know the elevation above still-water level at which this maximum occurs, and the height to which a completely obstructed wave acts. The maximum wave pressure occurs at a height, h_1, above still-water level, the theoretical value of which is:

$$h_1 = 0.785 \frac{h^2}{L} \qquad (17)$$

although better results are secured by:

$$h_1 = 0.12\ h \qquad (18)$$

When a wave is completely obstructed by a vertical surface of sufficient height, the wave crest is raised to a height equal to $h_0 = 2\ a$ above the still-water level, and, at this height, the pressure becomes zero. With the foregoing data it is possible to design the entire wave-pressure area and thus evaluate the total force to be resisted by a proposed structure.

(d) Oblique Waves.—Waves striking a breakwater obliquely produce less pressure than those that strike normally. Using the relations indicated in Fig. 2, the wave pressure, p, from Equation (12), is reduced to a value,

$$p_n = p \sin^2 \alpha \qquad (19)$$

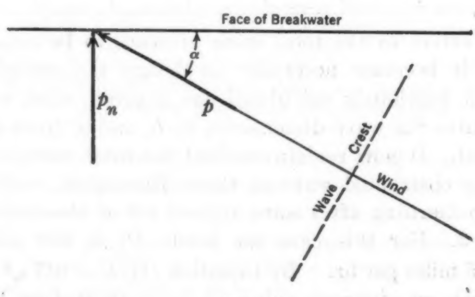

FIG. 2.

Observations made by the writer at Toronto, in 1915–1916, gave the following result: $p = 1\,460$ lb per sq ft; $p_n = 820$ lb per sq ft; and $\alpha = 50$ degrees. By Equation (19), $p_n = 1\,460 \times 0.587 = 857$ lb.

WAVE-PRESSURE OBSERVATIONS

(a) Wave-Pressure Observations on Lake Superior.—In 1901 and 1902 Captain Gaillard made numerous observations on Lake Superior which represent the best available data and will serve as a pattern for wave-pressure curves (see Table 4). It is regrettable that no accurate wind velocities were recorded, although it was stated that moderate wind, from 20 to 35 miles, produced these waves at Duluth, Minn. The pier carrying the dynamometers was not of sufficient height to obstruct the larger waves completely, so that for those higher than 13 ft the recorded pressures are for partly obstructed waves.

The coefficient, $\dfrac{k\,w}{2\,g} = 1.3$, in Equation (12), was chosen because none of the waves listed in Table 4 was caused by extreme storms, while the possible maximum coefficient, 1.71, given for Equation (12) is more probable for winds from 70 to 75 miles per hr with shorter waves.

TABLE 4.—WAVE-PRESSURE MEASUREMENTS, DULUTH CANAL, LAKE SUPERIOR

Date	Elevation, still-water level, in feet	OBSERVED MAXIMUM WAVE DIMENSIONS, DEPTH, d = 25 FEET			MAXIMUM DYNAMOMETER READINGS, IN POUNDS PER SQUARE FOOT			COMPUTED		
					Elevations of Dynamometers					
		Height, h, in feet	Length, L, in feet	Velocity, v, in second-feet	+0.07 ft.	+3.74 ft.	+7.01 ft.	$\frac{d_0}{L}$, by Equation (7)	v_0, in second-feet	p, in pounds per square foot
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1901:										
July 24	+1.7	12	150	24.2	250	1 150	1 030	0.180	7.8	1 330
Aug. 9	+1.9	12	130	24.2	370	1 190	780	0.210	8.0	1 348
Oct. 9	+1.9	13	150	29.6	0	1 615	1 260	0.180	8.4	1 876
Nov. 22	+1.4	14	150	27.2	0	1 605	1 605	0.184	9.0	1 705
Sept. 24	+1.9	16	250	33.2	1 630	2 255	2 050	0.110	9.3	2 344
					+7.04 ft.	+12.57 ft.	+16.18 ft.			
1902:										
Oct. 25	+1.7	16	200	30.0	1 755	1 335	0	0.138	9.5	2 026
Dec. 20	+1.7	16	210	31.0	1 700	1 430	515	0.131	9.4	2 120
Nov. 12	+1.7	18	250	32.0	2 370	2 195	1 370	0.110	10.5	2 344

In order to arrive at the total wave pressure to be resisted by a sea-wall or breakwater, it becomes necessary to design the complete wave-pressure curve from data previously calculated for a given wind velocity and fetch. This will determine the wave dimensions, h, L, and a, from which v, v_0, and p, may be evaluated. It now remains to find the total wave pressure for a completely or partly obstructed wave of these dimensions, and this must be approximated by patterning after some typical set of observations such as those shown in Fig. 3. For this case the fetch, D, is 260 miles, with a wind velocity, V, of 35 miles per hr. By Equation (1), $h = 017\sqrt{35 \times 60} = 16.5$ ft, as compared with an observed value of h = 16 ft (see Fig. 3). Furthermore, L = 210 ft; v = 31 ft per sec; and d = 25 ft. Then, $h - a = 5.18$ ft; $h_0 = 2a = 21.64$ ft; $\dfrac{d_0}{L} = 0.13$; $c = 0.81$; $\mu = 1.22$; v_0 (by Equation (16)) $= 7.11 \times 1.22 \times \dfrac{16}{\sqrt{210}} = 9.57$ ft per sec. The pressure, p_0 (= 1 600 lb per sq ft at a point midway between Points B and C, Fig. 3), may be assumed equal to 0.72 p when designing a wave-pressure curve.

By selecting the maximum pressure values for a 16-ft wave from Table 4 and plotting them as a composite pressure curve, the section shown in Fig. 3 was obtained. The crest of this 16-ft wave determined by Equation (4) (with C = 2), was 10.8 ft above the still-water level, and this crest was raised to a height of 18.3 ft for the partly obstructed wave, with the trough at 5.18 ft below the still-water level.

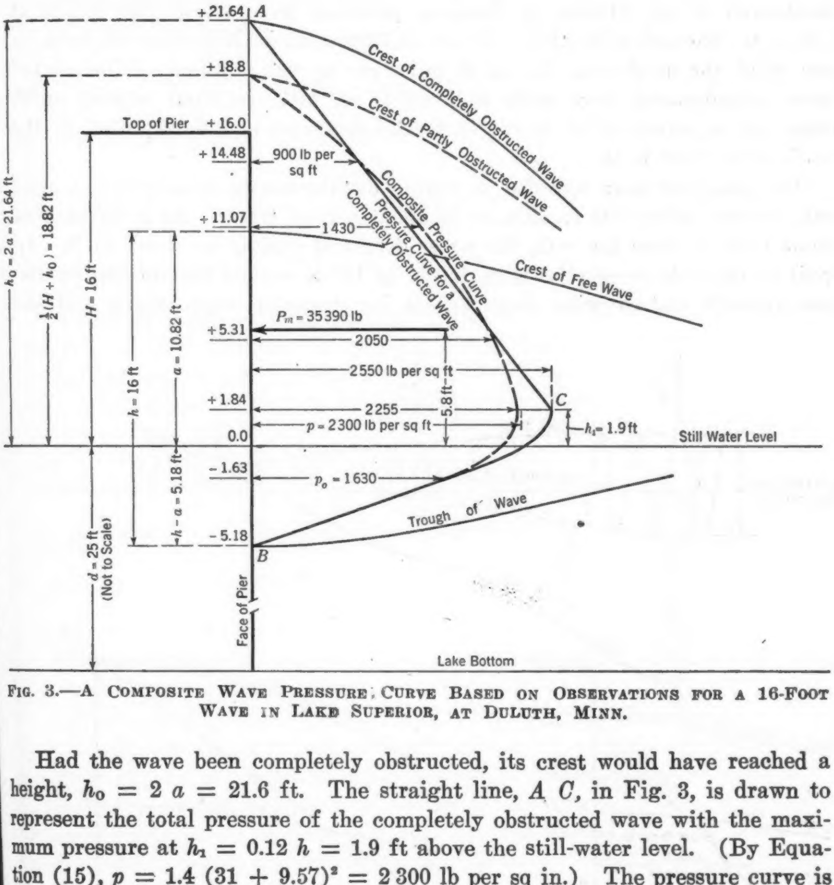

FIG. 3.—A COMPOSITE WAVE PRESSURE CURVE BASED ON OBSERVATIONS FOR A 16-FOOT WAVE IN LAKE SUPERIOR, AT DULUTH, MINN.

Had the wave been completely obstructed, its crest would have reached a height, $h_0 = 2\ a = 21.6$ ft. The straight line, *A C*, in Fig. 3, is drawn to represent the total pressure of the completely obstructed wave with the maximum pressure at $h_1 = 0.12\ h = 1.9$ ft above the still-water level. (By Equation (15), $p = 1.4\ (31 + 9.57)^2 = 2\ 300$ lb per sq in.) The pressure curve is completed by drawing the curved line, *C B*. The total wave pressure for the completely obstructed wave is represented by the triangular area, $A\ B\ C = P_m$ = 35 390 lb, acting at the center of gravity for the pressure area which is 5.8 ft above still-water level. For the partly obstructed wave as observed, the total wave pressure, *P*, is computed as 33 340 lb, acting at 5.42 ft above the still-water level.

The composite wave pressure curve thus described, represents the plotting of the maximum pressures recorded, for the duration of a storm, on the several dynamometers located at different elevations. It is evident that this is not produced by any single wave, but that waves of various sizes contribute to produce the maximum pressures recorded. Hence, the total pressure area of such a curve is undoubtedly greater than the actual and thus includes a certain factor of safety.

(b) Wave-Pressure Observations on Lake Ontario.—The record herein presented was observed by the writer in connection with the sea-walls and

breakwater of the Harbor of Toronto, produced by a storm that began at 5:00 P. M., November 18, 1915. At about 7:00 A. M., on November 19, with an east wind, the maximum, V_m, of 42 miles per hr was attained. Dimensional wave measurements were made at 10:00 A. M., with the wind velocity at 30 miles per hr, after which it veered to the southeast and finally died in the south about 2:00 P. M.

The pressures were recorded on spring dynamometers mounted on a solid crib situated about 700 ft offshore in about 8 ft of water. At a distance of about 1 200 ft from the crib, the water deepened rapidly to about 40 ft. In front of the crib, three piles were driven at 150-ft centers forming an equilateral triangle and carrying gauge-boards for observing wave height, and the

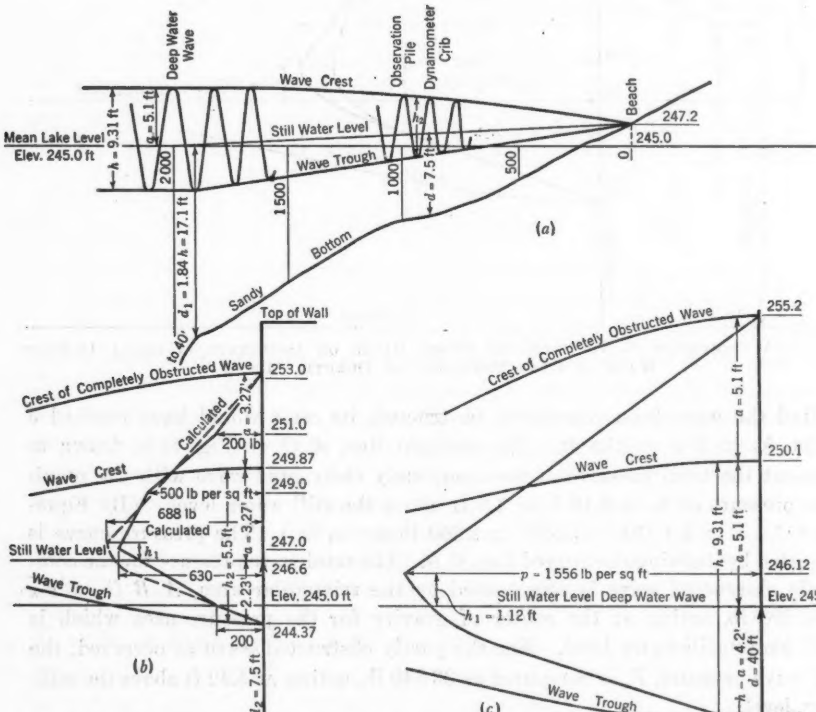

FIG. 4.—WAVE OBSERVATIONS ON LAKE ONTARIO, AT TORONTO, ONT., CANADA, STORM OF NOVEMBER 19, 1915, USING SPRING DYNAMOMETER

direction and velocity of wave crests. In Fig. 4(*a*) and Fig. 4(*b*), which show the results of these observations, the fetch, D, equals 156 miles from the east and 32 miles from the southeast. The wind velocity, V, was 40 miles per hr from the east, shifting to 30 miles per hr from the southeast, when the waves were at a maximum, and h_2 (Fig. 4(*b*)) = 5.5 ft; L = 118 ft; and v = 19.8 ft per sec.

By Equation (4) (with $C = 2$), $a = 3.27$ ft; $h_2 - a = 2.23$ ft; $h_1 = 0.212\ h_2$; $\frac{d_o}{L} = 0.068$; $c = 0.68$; and $\mu = 1.66$ (see Table 3). By Equations (7) and (14), $v_s = 16.72$ and $v_o = 6.28$, respectively. Consequently, $v_s + v_o = 23$ ft per sec, and, by Equation (15) (with $\frac{k\,w}{2\,g} = 1.3$), $p = 688$ lb per sq ft. Equation (8) gives $v_2 = 0.9\ v\ \sqrt[4]{\frac{7.5}{40}} = 17.45$ ft per sec. with a reduced height (according to Equation (9)) of $h_2 = 9.31\ \sqrt{\frac{7.5}{40}} = 6.14$ ft.

The deep-water wave dimensions in Fig. 4(*c*), were estimated for a wind velocity, *V*, of 30 miles per hr and a fetch, *D*, of 100 miles, representing something like an average condition prior to reaching the maximum. For this case, h (by Equation (1)) $= 9.31$ ft; $L = 21\ h = 195$ ft; $d_1 = 1.84\ h = 17.1$ ft; a (by Equation (4), with $C = 1$) $= 5.1$ ft; $h - a = 4.2$ ft; $h_1 = 0.12\ h = 1.12$ ft; $d = \frac{L}{5} = 40$ ft; $\frac{d_o}{L} = 0.22$; $c = 0.94$; and $\mu = 1.06$. By Equations (7) and (14), $v = 29.6$ and $v_o = 5.0$, respectively. Consequently, $v + v_o = 34.6$ ft per sec, and by Equation (15) (with $\frac{k\,w}{2\,g} = 1.3$), $\beta = 1\,556$ lb per sq ft. The still-water level was at Elevation 245.0 in deep water and began to rise when the depth was reduced to 17 ft where the waves broke, and finally reached Elevation 247.2 on the beach (Fig. 4(*a*)).

After entering shallow water, and producing the record shown by Fig. 4(*b*), the reduced wave had a height of 5.5 ft, a length of 118 ft, and a velocity of 19.8 ft per sec. These dimensions and pressures were checked by applying the foregoing equations to show the agreement between computed and observed values. It is thus seen that for a given condition of wind, fetch, and depth in which a proposed breakwater is to be built, the wave-pressure curve may be estimated with sufficient accuracy to test the stability of the proposed structure.

Stability of Breakwater Cribs Subjected to Wave Force

The foregoing parts of this paper present the several steps in evolving a wave-pressure curve from a given fetch and wind velocity, or, in other words, of evaluating the wave effect produced by a certain cause. The ultimate aim, however, is to design a breakwater crib of adequate dimensions to withstand, safely, the wave pressure to which it may be exposed during the most severe storms.

The complete solution of such problems will now be undertaken on the assumption that the exact location for a proposed structure and the maximum wind for the locality are known definitely. The examples chosen are the breakwater at Harbor Beach, Mich., and Magann's Pier at Toronto, both of which have withstood many severe storms since 1900 without showing any

marked structural weakness. Hence, if the method of analysis herein given leads to findings in conformity with the actual behavior of these structures, the speculative features involved in the method will have been reduced to a negligible quantity. In this connection the wind roses in Fig. 5 and the weight data in Table 5 will be of value. The wind roses (Fig. 5) show maximum wind velocities for the eight cardinal points and the dates on which they occurred. These data were obtained from the officials of the U. S. Weather Bureau at the several stations. All the wind velocities are based on readings from three-cup anemometers.

TABLE 5.—WEIGHT DATA FOR DESIGNING BREAKWATER CRIBS

Material	Weight, in Pounds per Cubic Foot		Coefficients of Friction	
	In air	In water	Materials	Coefficient
Granite blocks	170	107.5	All surfaces of masonry or brick-work in contact	0.65 to 0.70
Limestone blocks	166	103.5	Stone or brickwork on moist unctuous clay	0.30
Limestone rip-rap (40% voids)	100	62.5	Stone on bed-rock, dry	0.70
Concrete, plain	148	85.5	Concrete blocks on well wetted concrete floor	0.70
Concrete, reinforced	152	89.5	Concrete blocks on rubble base, submerged	0.65
Timber, green, as shipped	35	27.5*	Rubble-filled cribs on rubble mound, submerged	0.90
Timber, wet (including bolts)	50	12.5*		
Average timber cribs, filled with limestone	92	50	Rubble-filled cribs on bed-rock submerged	0.60

* Uplift.

Problem 1.—Stability Investigation of Harbor Beach Breakwater (See Fig. 6).—The fetch, D, for Harbor Beach Breakwater, scaled from a map of Lake Huron, is found to be 132 miles north, or 92 miles northeast. The latter fetch is about normal to the breakwater. The maximum wind velocity from the north is taken as 60 miles per hr.

Since waves usually swing more or less normally to the shore as the water becomes shallower, the larger wave is assumed to arrive normally in front of the wall. Hence, the deep-water wave by Equation (1) has $h = 15.13$ ft and by Equation (3), $L = 14.0\ h = 212$ ft. The height of this wave above still-water level by Equation (4) (with $C = 1$) is $a = 8.65$ ft. making $h - a = 6.48$. The deep-water wave will break when the depth becomes $d_1 = 1.84\ h = 27.84$ ft. Since $d = 28$ ft, the wave will strike the breakwater before breaking, and hence without appreciable reduction due to shoaling effect. The problem is thus simplified.

By Equation (7), $\frac{d_0}{L} = \frac{28}{212} + 2\left(\frac{15.13}{212}\right)^2 = 0.142$, for which Table 3 gives $c = 0.829$ and $\mu = 1.21$, by interpolation. Equation (16) now gives $v + v_0 \doteq 36.2$ ft per sec, and the maximum unit wave pressure by Equation (15) (with $\frac{kw}{2\,g} = 1.7$) becomes $p = 2\,230$ lb per sq ft, acting at $h_1 = 0.12\ h$

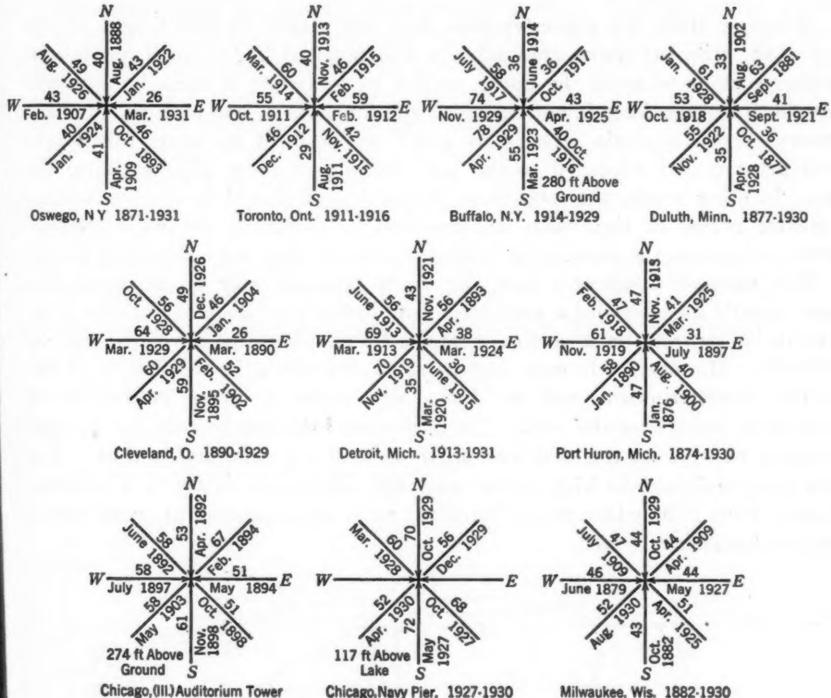

FIG. 5.—WIND RECORDS FOR THE GREAT LAKES

= 1.82 ft above still-water level. Then, make $p_0 = 0.72\ p = 1\,600$ lb per sq ft, located midway between the wave trough and the maximum pressure ordinate. These data determine the complete wave-pressure curve shown in Fig. 6, which curve requires only slight modification for the portion of wave that is not obstructed by the superstructure.

The total effective wave pressure is thus found as $P = 27\,424$ lb per lin ft of crib and acts 31.3 ft above the bottom. The weight of the crib, allowing for buoyancy, is calculated as $G = 95\,150$ lb per lin ft, which combined with P gives the resultant, R, acting on the base and producing a maximum unit pressure, $f = \dfrac{2 \times 95\,150}{3 \times 9.8} = 6\,470$ lb per sq ft at the heel of the crib. No account was taken of the water on top of the superstructure.

The safety factor against overturning is then, $\dfrac{95\,150 \times 18.82}{27\,414 \times 31.3} = 2.08$, and against sliding, $\dfrac{95\,150 \times 0.5}{27\,424} = 1.74$. Referring again to Fig. 6, it will be noticed that the wave force, P, was combined directly with the weight, G, in testing the safety of the structure. This would be correct in testing the stability of the crib, provided the entire wave impact is imparted simultaneously to all parts of the structure.

However, since the superstructure does not attain its full height at the face of the crib, but slopes gradually for a distance of 18 ft, and then presents another surface to resist the upper portion of the wave, it must be apparent that the full wave force is not expended simultaneously, nor with the pressure assumed in the analysis. The wave could not be built up to its full height until after it had advanced to the last obstruction 1 sec after striking the first. In other words, the total wave energy is transferred to the crib during a greater period of time than was assumed in designing the wave-pressure curve, and, hence, the shock is in reality less severe than the calculation shows.

This example illustrates how the wave pressure may be rendered less severe simply by adopting a superstructure design that does not obstruct the wave in its entirety on a vertical surface, by allowing the wave to build up gradually. However, this may develop some difficulties in the design of the concrete superstructure, and no doubt will cause a larger proportion of the wave to jump over the wall. The back-wash may also be retarded to such an extent as to meet the next oncoming wave at an unfavorable time. The wave force striking the high center wall will subject the latter to a moment of about 9 600 ft-lb, which might cause the mass concrete deck to break unless steel reinforcement is used.

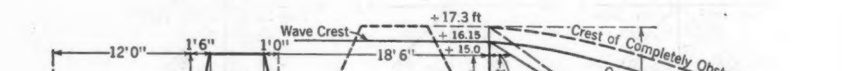

FIG. 6.—STABILITY INVESTIGATION, BREAKWATER, AT HARBOR BEACH, MICH.

Considering the ample safety of the crib itself, it would appear that the high mass concrete wall might better have been placed over the lake front, thus increasing the resistance to overturning, and affording greater security for the concrete face blocks, besides adding materially to the strength of the deck where it receives the worst punishment.

In designing a breakwater superstructure, much depends on the purpose it is to serve. It may be necessary to obstruct oncoming waves completely, especially if ships are intended to moor on the harbor side. In other cases, it may not matter how much water is thrown over the wall as long as the wave force is rendered harmless. The superstructure of the Harbor Beach Breakwater, if intended to protect ships while tied on the harbor side, might have been designed as indicated by a dotted outline in Fig. 6, so as to obstruct more completely the oncoming waves, at the same time to add resisting moment to the crib and increased strength to the deck. The use of some reinforcement in connection with these superstructure designs is very desirable.

It is also necessary to provide "breathing" openings through the concrete deck to allow relief for the air pressure formed in the body of the crib. Wave

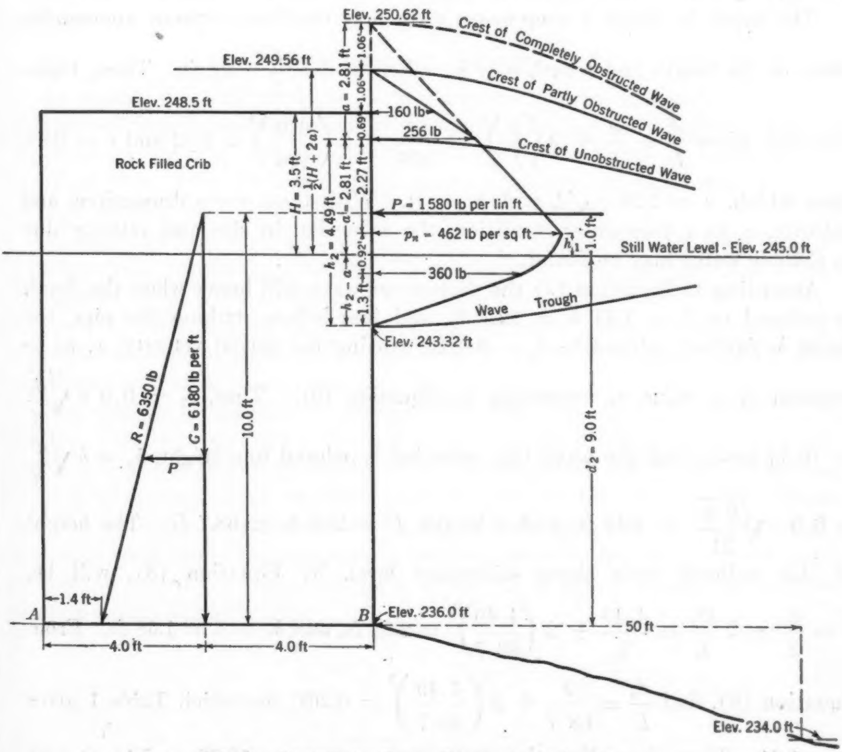

FIG. 7.—MAGANN'S PIER, TORONTO, ONT., CANADA

pressure will also set up hydrostatic pressure on the inside of the cribs, thus exerting a bursting effect on every part of the structure, all of which must not be overlooked.

Problem 2.—Stability Investigation of Magann's Pier.—This pier (see Fig. 7) is located between Jameson Street and Dowling Avenue, in Toronto, in 9 ft of water, deepening to 11 ft at a distance of 50 ft offshore. The worst condi-

tions exist for a southeast wind with a fetch, $D = 30$ miles, making an angle, $\alpha = 68°$, with the lake face of the pier. A wind velocity of 55 miles per hr was taken as a possible maximum.

This problem involves all the complications of an inshore sea-wall, and differs from the previous case in that the deep-water wave will encounter shallow water before striking the pier, besides approaching the pier obliquely instead of normally. Also, the pier is not high enough to stop completely the oncoming wave. Hence, the deep-water wave suffers a reduction in height, is only partly obstructed, and the final reduced pressure acts only by the amount of its normal component. The stability investigation for the fetch and wind velocity previously noted, may proceed as follows: For $D = 30$ miles, $V = 55$ miles per hr, the deep-water wave, by Equation (1), will have a height, $h = 0.17 \sqrt{V D} = 6.90$ ft, and by Equation (2) find $L = 15.3\ h = 106$ ft.

The depth in which a deep-water wave may continue without appreciable effect on its height and length may be taken as $d = \frac{L}{5} = 21$ ft. Then, Equation (8) gives $\frac{d_0}{L} = \frac{d}{L} + 2\left(\frac{h}{L}\right)^2 = \frac{21}{106} + 2\left(\frac{6.9}{106}\right)^2 = 0.33$ and $c = 0.98$, from which, $v = 2.26\ c\sqrt{L} = 22.8$ sec-ft. With these wave dimensions and velocity, v, as a deep-water condition, the reduction in size and velocity due to shallow water may be found.

According to Equation (5) the deep-water wave will break when the depth is reduced to $d_1 = 1.84\ h = 12.7$ ft, and just before striking the pier, the depth is further reduced to $d_2 = 9.0$ ft, causing the initial velocity, v, to be retarded to a value, v_2, according to Equation (9). Thus, $v_2 = 0.9\ v\sqrt[4]{\frac{d_2}{d}} = 16.53$ sec-ft, and the wave thus retarded is reduced to a height, $h_2 = h\sqrt{\frac{d^2}{d}} = 6.9 \sqrt{\frac{9.0}{21}} = 4.49$ ft, with a length, $L = 15.3\ h_2 = 68.7$ ft. The height of this reduced wave above still-water level, by Equation (3), will be, $a = \frac{h_2}{2} + 2\frac{h_2^2}{L} = \frac{4.49}{2} + 2\left(\frac{4.49}{68.7}\right)^2 = 2.81$ ft, and $h_2 - a = 1.68$ ft. From Equation (8), find $\frac{d_0}{L} = \frac{9}{68.7} + 2\left(\frac{4.49}{68.7}\right)^2 = 0.261$, for which Table 1 gives $\mu = 1.04$. Then, from Equation (12), find $v_2 + v_0 = 16.53 + 7.11 \times 1.04 \times \frac{4.49}{\sqrt{68.7}} = 20.5$ sec-ft.

The maximum unit wave pressure then becomes (by Equation (15), with $\frac{k\,w}{2\,g} = 1.30$): $p = 546$ lb per sq ft, acting at $h_1 = 0.12\ h_2 = 0.54$ ft above still-

water level. This pressure resolved normally to the pier as in Equation (19), becomes $p_n = 546 \times 0.846 = 462$ lb per sq ft. From the foregoing data the final reduced pressure curve, shown in Fig. 7, was obtained, giving the total wave pressure, $P = 1\,580$ lb per lin ft of pier. The weight of the crib, allowing for buoyancy, is $G = 6\,180$ lb per ft, which, combined with P, gives the resultant, R, acting on the basal plane, $A\ B$. The maximum unit pressure at the heel of the crib is, $f = \frac{2 \times 6\,180}{3 \times 1.4} = 2\,950$ lb per sq ft. Similar to Example 1, the factor of safety against overturning is, $\frac{6\,180 \times 4}{1\,580 \times 10} = 1.57$; and, against sliding (assuming a coefficient of friction of 0.5), is, $\frac{6\,180 \times 0.5}{1\,580} = 1.95$.

Concluding Remarks

In closing this subject the writer wishes to emphasize that stability investigations of this character, or designs based on similar calculations, must be regarded as more or less speculative; yet the methods herein presented for the first time, were developed after much thought and study, and are considered as accurate as the subject warrants.

This method of analysis was applied to existing breakwater structures at Presque Isle, Ontonagon, Grand Marais, Agate Bay, Portage Lake, and Marquette, all on Lake Superior. The breakwater at Buffalo, N. Y., comprising several designs with varied exposures, was also analyzed. In each case, the results of the analysis agreed closely with the behavior exhibited by these structures during many years of service.

The western breakwater at Toronto, built in 1915 and 1916, according to designs that were severely criticized at the time as being structurally inadequate, showed a factor of safety of about 4 against both overturning and sliding, according to an analysis made in 1915 by the writer. This breakwater, covering a length of about $3\frac{1}{2}$ miles, has weathered all storms for the eighteen years since its construction without manifesting the slightest signs of weakness, again demonstrating the reliability of results obtainable with the method herein described.

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DISCUSSIONS

HIGH DAMS ON PERVIOUS GLACIAL DRIFT

Discussion

BY MESSRS. HARRY H. HATCH, AND EDWARD M. BURD

HARRY H. HATCH,[25] M. AM. SOC. C. E. (by letter)[25a].—It may be inferred that the author is belittling, unintentionally, the importance of good rock foundations for earth dams. The fact that 100 ft of head against an embankment 120 ft high on a foundation of pervious glacial drift is considered an accomplishment, should in no way classify the Hardy Dam as a high earth dam. For really high earth dams it is still necessary to follow the Biblical injunction and the caution taught by past and present experience.

The problem is a question of permanency and the permissible quantity of seepage both from viewpoints of its economic value and its effect on the safety of the structure. With his wide experience with this kind of foundation and embankment, the author has evidently solved the problem to the satisfaction of his requirements. This solution, however, should not be considered as of general application, but should be restricted to the same type of foundation, similar embankment material, the same height of dam, the same head of water, and the same economic conditions; and the same care should be exercised in construction.

It has been shown that, approximately, seepage varies: Directly as the square of the effective size; directly as the sixth power of the percentage of voids of the core material; directly as the square of the head; and, inversely, as the path of percolation. At Hardy Dam the loss of water by seepage may not have any economic value, but the same quantity may become a big factor elsewhere, and with a higher head, or for other reasons, the increase in seepage may prove fatal to the structure.

Any difference in settlement, whether in the foundation or in the embankment, involving the relative superimposed load, is due to the difference of the percentage of voids of the respective materials, and also to the differ-

NOTE.—The paper by Edward M. Burd, M. Am. Soc. C. E., was published in April, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: May, 1933, by Messrs. Charles W. Sherman, and F. B. Marsh; September, 1933, by M. M. O'Shaughnessy and H. de B. Parsons; October, 1933, by Messrs. Joel D. Justin, and A. K. Pollock; and November, 1933, by Messrs. William P. Creager, Adolph J. Ackerman, J. Albert Holmes, and Irving B. Crosby.

[25] Engr. in Chg., Cobble Mountain Reservoir, Springfield Water-Works, Blandford, Mass.

[25a] Received by the Secretary December 26, 1933.

ence in their moisture content. Under a given load there is a limiting consolidation or a minimum percentage of voids in any given material. This consolidation takes place very rapidly at the beginning of the application of the load; then very slowly. The time necessary for ultimate consolidation will depend on the quantity of water to be squeezed out during the re-arrangement of the material particles from initial to final percentage of voids, or during their adjustment to final structural equilibrium under the maximum load. This also depends on the path of percolation, head of water, and particle sizes.

It would be desirable if the author itemized the 30-cent cost per cu yd with maximum and general average yardage per working day, analyzing the cost per yard in terms of: (*a*) Excavation; (*b*) conveyance to the dam site; and (*c*) placing in the dam.

It would be interesting to know how the author determined the position of the fineness gradation lines in Fig. 6. For instance, the 1.00-mm line which shows that almost 100% of the sluiced material is larger than 1.00 mm at the bottom of the embankment, and, at the bottom of the top trestle, it shows that almost 100% of the sluiced material is smaller than 1.00 mm. The intersection of the 1-mm line with the slopes of the sluiced material at various elevations is not consistent, and yet in sluiced beaches of a dam the gradation of particle sizes is almost uniform, varying from the largest at the top of the slope to the smallest near or in the pool. If the fineness gradation lines in Fig. 6 are based on Hardy Dam, or similar, materials, Fig. 5 (curves for Hardy Dam) fails to show such a range of variation in the particle sizes of the borrow-pit deposits that would satisfy the conditions of Fig. 6.

EDWARD M. BURD,[26] M. AM. SOC. C. E. (by letter)[26a].—One of the discussers, Mr. Sherman, criticizes the writer's generalization concerning a "1 on 5 slope from head-water to tail-water," as being inconclusive without more exactly defined conditions. Perhaps that generalization is inconclusive if it is not read with the presumed limitations explained in the text immediately preceding that reference. Evidently, Mr. Sherman desires, in common with every one, a more exact determination of each problem of percolation, which then could be solved much more precisely by known laws. As was brought out in the paper, however, glacial deposits are too heterogeneous to permit of such treatment, at least within economic limitations. Consequently, rule-of-thumb, but safe, generalizations for the existing materials were suggested, and it is hoped will be understood with the stated reservations.

The four examples of hydraulic gradient cited by Mr. Sherman are excellent examples of the range of earth dams in general, supplemented perhaps by the 1 on 10 or the 1 on 12 slopes common in "silt" dams as ordinarily built in India, representing the extreme of flatness. The second example cited is representative of materials and conditions usually encountered in the heavily glaciated area, as economically applied to the problem.

[26] Civ. and Hydr. Engr., Consumers Power Co., Jackson, Mich.
[26a] Received by the Secretary April 2, 1934.

The writer approves and acclaims whole-heartedly the remarks of Mr. Marsh, as evidence of a common-sense understanding of the practical, as well as the theoretical, problem. The possibilities of steel sheet-piling are far from understood; the subject is far from being exhausted. In this connection the recent standardization, eliminating many of the best and useful sections, is somewhat disheartening to those forced to deal with deep sand and gravel driving.

Answering Mr. O'Shaughnessy's suggestion of more details of design and construction of concrete core-walls, in addition to the data in Paragraph 6 under "Design and Settlement of Embankments," the usual thickness might well be given. The top 20 ft is frequently made 12 in. thick, increasing in thickness about 6 in. for each successive 20-ft depth. Thickness, however, seems of secondary importance. During construction a difference in water head on either side of any observed wall shows that leakage occurs at joints, form ties, occasional stone pockets, etc. The generally observed decrease in percolation with lapse of time may well be due to silting and plugging of such minor, but numerous, points of leakage.

As pointed out by Mr. O'Shaughnessy, permissible limits of leakage vary widely with geological structure and economy. In glacial drift it surely would be economically impractical and probably physically impossible to limit leakage to rates cited for the Hetch Hetchy Dam. Based on past experience, each new project is considered on its economic merits, rather than on a basis of safety. Safety is a major consideration, of course, but safety limitations in glacial structure usually far exceed the economic balance. As stated in the paper, one dam similar to Hardy Dam, but without a core-wall, safely contains an initial leakage approaching 20 cu ft per sec. The glacial problem is essentially economic.

As to hydraulic pumping of embankment materials, here, again, generalizations are unwise. As far as can be determined, no floating equipment has ever been known to build an embankment in the glacial region under discussion. The largest operators of such equipment had every opportunity to bid on the Hardy Dam embankment, but could not compete on a cost basis with mechanical transportation.

It is difficult to understand how Mr. O'Shaughnessy can "confute" the actual measured settlement of the Hardy Dam embankment, by what may have happened at the Priest Dam, probably under wholly different conditions. He forgets—and the error is quite common—that the writer is discussing solely glacial-drift materials and conditions; and perhaps the major difference between washed glacial-drift embankments and the more usual earth dams lies in the considerably smaller settlement of the former, and its relatively quick cessation as soon as the pond is filled. As compared with this condition, the usual earth dams of equal height settle at least three times as much, extending over a term of years. The latter condition probably applies to the Priest Dam.

Mr. Parsons brings up the very active question of steam-hydro-electric operation as regards firm and peak power. On a system roughly "half and half," of which Hardy Dam is a part, it is much better to put steam on the

base, and build the "hydros" as daily peak plants; that is, 8 to 10 hr per day, $5\frac{1}{2}$ days per week. While generalizations are proverbially dangerous, it would seem that "hydro," in general, is being used more and more on a peak basis, except during high water when it would be wasted if not used continuously.

The old valley notch which Mr. Parsons describes as under Spier Falls Dam, is most interesting, the condition being a common one in glacial valleys, and the necessary treatment being usually a matter of judgment. In Michigan, such material has of necessity been left in place and used as foundation; apparently, as long as it is covered and loaded, it maintains its structural integrity.

As for "quicksand," the writer would never take issue with Mr. Parsons' long experience in these matters, and recognizes the many variations and gradations in such materials of "quick behavior." From personal experience, the writer can corroborate Mr. Parsons' complete success in settling boiling "quicksand" by adding a layer of crushed stone, actually purchased for concrete aggregate. A layer only a foot thick will completely transform a boiling-spring bed into which a man would quickly disappear, into a reasonably stable foundation which will immediately support loads in the order of 1 ton per sq ft.

Mr. Parsons' cautions about dependability and tightness of concrete face paving are well taken. However, they apply practically unchanged to central core-wall construction also. In either case, good engineering and good construction are essential. The writer here suggests that face paving of concrete might well be water-proofed by some flexible, jointed metal, mopped fabric, or similar membrane treatment; or what would be equally effective, mud directly on the face of the more pervious earth fill.

The writer appreciates sincerely the discussion contributed by Mr. Justin. In this paper, it has been the writer's intention merely to discuss in greater detail one particular condition and variation of earth dam construction, as very ably covered in textbooks and other publications by Mr. Justin, whose comments upon the economics of the situation as regards steam *versus* hydro-electric power surely are instructive and pertinent and, as he states, contrary to considerable general belief and some prejudice. There seems to have been a noticeable improvement in the general operating attitude on this subject within the last few years.

In common with several others, Mr. Justin has corrected a rather too general statement in the paper concerning relation of fineness of embankment material and slope of the line of saturation. For fine materials beyond the range of sands encountered in the glacial area, it is common practice, of course, to build more nearly impervious embankments of much steeper saturation slope. Perhaps the writer should have been more specific in limiting his statements to the relatively porous and quick-draining sands economically obtainable from glacial deposits, and with other conditions as normally encountered in the region under discussion.

As regards impervious up-stream layer or membrane compared with the more conventional core-wall, Mr. Justin's comments are quite apt and are

made with a clear understanding of the practical difficulties involved with each form of construction. Years of experience with the core-wall both in design and construction, are leading the writer more and more to favor the up-stream form of water cut-off. While it is true that there have been some failures with the earlier forms of this design, it is likewise a fact that there have been failures with central core-walls and all other types of construction which, of course, must be designed on the basis of experience as well as theory. Engineers of the U. S. Reclamation Service apparently have solved successfully the problem of up-stream cut-off and have corrected the earlier difficulties, which were relatively minor, as no major failures were ever chargeable to these difficulties. As far as concerns the usual fear of subsequent settlement of embankment materials, it was one of the writer's major considerations in preparing this paper to show that after several years of close observations on many level points it was proved beyond any refutation that properly placed and water-laid glacial sand embankment materials were notably free from such continuing settlement, as directly contrasted with the usual form of earth embankment on which an up-stream concrete blanket would obviously be an unsafe and impractical form of construction.

On the other hand, a concrete slab is not necessarily the best solution and for many conditions the writer wholly agrees with Mr. Justin that an up-stream blanket of fine material probably is even better, and usually would be less expensive. In general, it is not the form of the up-stream cut-off that is of so much importance as the contention that it should be placed on the up-stream face rather than through the center of the embankment, all other conditions being equal. As for rapid drawdowns of ponds of the size contemplated by present-day projects, there would seem to be but little occasion for any rapidity of the degree necessary to cause unbalanced upward hydrostatic pressure on any up-stream slab construction, because ordinary operation normally draws ponds very little, and slowly. Any disastrous failure, of course, might draw a pond rapidly, but in that case its effect would be of minor consideration as compared with the event itself. The writer only hopes that the paper and its discussion may awaken more active interest in this pertinent design problem as affecting earth embankments.

Correctly, Mr. Pollock points out that the more conservative percolation factors advocated by Bligh (who, perhaps, deserves credit for first developing this idea thoroughly), have proved to be more conservative than necessary, under the particular circumstances of the paper in question. However, this should not detract from the excellence of Bligh's pioneering and, moreover, the writer's generalization probably is limited to the peculiarities of the glacial area. The writer is far from satisfied with the state of knowledge of this matter, in general, and has offered his paper in the hope of developing more information and keener interest. It would seem as if the importance of progress in earth embankment construction, both as regards safety and economy, would justify more intensive study and experimentation with the more porous materials than has been made to date. The relatively impervious clay core type of embankment has been developed and proved quite thoroughly

under the leadership of such experts as A. S. Crane, M. Am. Soc. C. E., for example; but similar development of the more porous materials and perhaps with even greater economic advantage still awaits the time, effort, and financial resources of adequate study. For this reason, the writer would hesitate to put forth any generalization or general application of percolation factors developed for this rather limited condition under discussion, which by no means covers the entire general field.

Mr. Pollock raises the question of bulking of sand when moderately moist; that is, a condition intermediate in moisture content between the wet condition as first placed and the dry condition such as prevails later. If there is any evidence of this peculiarity, it has escaped notice on the many construction jobs involving sand embankments of the kind described in the paper. In the ordinary construction process—that is, with water being fed to the top layers at close intervals in order to wash them to place—the material in the embankment probably never dries out to the extent of reaching the bulking point. After the embankment has been completed, however, a time must come when that portion above the saturation line would pass through the bulking range of moisture content. If any such bulking action takes place at Hardy Dam, it has escaped notice as such. Check levels are still taken (1934) about twice a year along the full length of the crest wall (which is the upper part of the core-wall construction); but to date there has been no further change, either up or down, since the initial settlement described in the paper. Possibly the weight of the structure would prevent any volume increase, except close to the surface where it would pass unnoticed.

In washing the embankment material into place, it has been observed many times that the wash water strikes down through the pervious sand very rapidly. Frequently, a large monitor stream will lose itself in a few hundred feet, much as streams in the Western Plains area will drop out of sight at the more pervious reaches of their stream beds. Even while the washing stream is carrying sand, the material immediately beneath it is so hard that the workmen walk all over it at all times, and within a few minutes after an area of sand has been washed to place, and the water has disappeared from the surface, the settlement is so complete that an automobile can be driven without difficulty over the new fill. Obviously, this method of placement secures minimum volume and maximum compactness for the fill of material and a few measurements made in times past have indicated that the volume for a given mass is appreciably less than that occurring in the glacially washed and placed borrow-pits from which the material is taken in the natural state. Whether or not this condition has any bearing upon the bulking situation is unknown to the writer, but at least no indication of volume increase has ever been indicated by the many levels run over the reference points at all times during and subsequent to construction.

Mr. Pollock questions the interpretation of the settlement observations, and apparently the writer has not made the details sufficiently clear. The core-wall footing is centered over the steel sheet-pile cut-off, but is provided with a slip joint so that it can settle down over the top of the sheet-piling without

the latter offering any appreciable supporting resistance. This arrangement is considered necessary as a precaution against the core-wall "hanging up" on the piling and possibly leaving unfilled spaces beneath the footing which might form dangerous water passages. Consequently, the full weight of the core-wall is carried directly on the natural and undisturbed earth formation on which it was built, and the core-wall settlement record, which includes the crest wall record also, represents essentially the subsidence of the undisturbed glacial formation of the valley floor. There might be a minor but negligible compression shortening of the core-wall height.

Referring to Fig. 4, the settlement record noted as "Total Settlement of Crest Wall" represents the results of levels taken at a number of points directly at the top of the main vertical core-wall to about December 1, 1930, by which time the bulk of the embankment was all in place. Then, as the pond was filled and the crest wall was constructed, as indicated, in February, 1931, it was necessary to transfer the reference points at the top of the main vertical section of the core-wall to other reference points at the top of the crest wall construction, which obviously would settle with the embankment upon which it was placed. This curve then, beyond December 1, includes some embankment consolidation, noted and estimated to be about one-half; but the 3 in. of total settlement to December 1, 1930, must represent, to all practical purposes, only the subsidence of the valley floor. The penstock settlement curve of the same figure represents reference points on the top of the base slab of the penstock; hence, except for negligible consolidation of the concrete in this base slab, it must represent also the general subsidence of the valley floor, which under the penstock was $2\frac{5}{8}$ in. as of December 1, 1930. The penstock section was constructed on a fairly thick layer of mudstone which presumably spread out and decreased the intensity of loading, with consequent slight lessening of settlement over that represented by the more highly concentrated loading directly under the footing of the core-wall. At least, this would seem to be the probable explanation of this slight difference.

The loading intensity at the bottom plane of the base slabs of both the penstock and the core-wall was computed to be practically the same. The curve of power-house settlement also represents valley floor subsidence except for negligible consolidation of the heavy reinforced concrete construction. The fourth curve, however, noted as "Embankment Consolidation at Deep Section," represents essentially only the consolidation of the embankment section itself, after it was carried to full height. The embankment fill, of course, consolidated during its entire construction, but there was no practical way of measuring this consolidation during the building process, nor any practical consideration that would make it desirable. Only the consolidation after completion to full height, as shown by the curve, would have a bearing on the safety and wisdom of placing construction directly on the surface of the completed fill, which was one of the main questions raised by this paper.

The writer appreciates Mr. Creager's willingness to compare Soft Maple Dam and Hardy Dam because of the similarity of natural conditions, with a difference in methods of construction. Presumably, both methods have their

good points, but the combination and comparison offers much interesting and valuable information. The writer visited Soft Maple Dam several times in order to study the formation on which it was built and he agrees with Mr. Creager that most of the very considerable seepage at Soft Maple Dam is presumably passing around and under the dam through the peculiarly porous glacial drifts, rather than through the dam structure itself. The writer further agrees that the effectiveness of the Hardy type of core-wall in the Soft Maple Dam would be questionable; and probably the cost of it would scarcely be justified by the decrease in total seepage. It would seem apparent from the paper that under such conditions an up-stream type of cut-off would have advantages over the vertical core-wall and the writer agrees that an up-stream blanket of fine material would be more effective at Soft Maple Dam than a core-wall. He is rather inclined to agree also with this up-stream treatment for the usual glacial formation; but as yet no conclusive large-scale demonstration of this method has been undertaken, although it has been used effectively on a small scale on Hardy Dam, and on earlier construction in Michigan. At the Soft Maple Dam site Mr. Creager was faced with a difficult situation which apparently was well handled and nothing in his discussion of the paper was intended as any criticism whatever.

Mr. Ackerman's comments are valuable and practical. If his classification chart (Fig. 7) and definition were used consistently, it undoubtedly would be an improvement to the nomenclature and understanding of earth embankments. As he points out, much confusion has arisen through lack of specific designation when discussing the subject.

As for the ice-thrust considerations upon the design of the intake tower at Hardy Dam, the writer might add that in the latitude of Northern Michigan, ice thrust can and does become a major factor. The intake tower at Hardy Dam was designed to resist that thrust, which might be encountered and which, in this case, would be absorbed to some extent by the fill around the lower portion of the tower, while the bridge to the top of the tower was designed to carry its portion of the thrust back to the embankment proper. As for the side thrust, the shape of the tower is such as to be well braced against side action.

Mr. Holmes' description of ground-sluicing methods by the Northern Pacific Railway Company parallels in economy and effectiveness similar sluicing in the construction of the earlier dams in Mighican under the direction of Mr. William G. Fargo. The first sites developed were selected, to a considerable extent, with a view to direct sluicing from the adjacent high hills and much of this yardage was placed at costs then prevailing in the order of 7 cents per cu yd. Unfortunately, in later developments, the engineers have not been able to avail themselves of topography that would make such sluicing possible, with the result that it has been necessary to move the fill by other and more expensive methods. Where direct sluicing is possible, however, it is believed that no other method has ever approached it in economy and general suitability.

Regarding settlement, Mr. Holmes offers most interesting data relative to the more recently built Fifteen-Mile Falls Dam on the Connecticut River.

It is regrettable that similar comprehensive methods of making subsequent settlement measurements available to the profession have not been more widely adopted in dam construction. It will be a number of years, of course, before the final settlement facts of this relatively new dam will be available for use. The final statement of Mr. Holmes that the total settlement in a well rolled embankment is not great, would seem to be somewhat indefinite and perhaps difficult to prove. Fortunately, he has elsewhere contributed[27] perhaps the best available record of settlement of typical sluiced-center, so-called semi-hydraulic types of embankment, being the records for Somerset and Davis Bridge Dams in New England. The writer begs to repeat the generalities of these data at this point in order to show how the settlement records compare with the experience of the sluiced sand dam at Hardy where the 115-ft maximum section of embankment material consolidated only $1\frac{1}{4}$ in. after being topped out and has shown no further consolidation whatever for about three years since the pond was filled. As compared with this, the record of the Somerset Dam for a maximum height of 105 ft shows[27] a settlement of almost 1 ft in the first 15 yr after construction and even at that time continuing at the rate of approximately $\frac{3}{8}$ in. per yr; although this rate is evidently approaching a point of equilibrium at an age of perhaps 25 yr.

The same record shows that the Davis Bridge embankment at the end of the first 5 yr had settled about 0.6 ft in the section approximately 200 ft deep and the similarity of the curves would seem to indicate about the same expectancy. Thus, there appears to be a fundamental difference between the long-continued characteristic settlement of the more impervious earth-type construction as compared with the immediate and complete settlement of much less total magnitude for the water-laid sand dam. This difference is of general observation, although not so exactly measured at other dams; at least the data have never been made available. However, Mr. O'Shaughnessy calls attention to a settlement of $2\frac{1}{2}$ ft of the up-stream fill of Priest Dam, in California, an earth structure 145 ft high, and a 6-ft settlement of the downstream fill of the same structure. Noticeable and evident settlement of this magnitude has been observed by the writer on another semi-hydraulic sluiced core dam of compressible materials of a height of about 170 ft, which was completed several years ago.

The writer acknowledges the correction suggested by Mr. Holmes in connection with the lowest curve on Fig. 4 which, on further thought, should have followed the dots as undoubtedly representing what actually took place rather than as indicated by the smooth rounding off of the curve to which Mr. Holmes quite properly takes exception.

It is always helpful to the Engineering Profession to have the advantage of comments from geologists, who have a further removed perspective on some of the problems treated in this paper, and who, of course, have the advantage of a better understanding of the formation of the site and adjacent region. Technical geological assistance has been of great value in studying these problems in the complicated glacial formation, and it would seem as if a

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[27] *Engineering News-Record*, November 14, 1929.

thorough geological study should and must precede any safe and well-organized program of dam construction. For this reason and because of the general information added to the paper, the comments by Mr. Crosby are thoroughly appreciated.

Several of the questions raised by Mr. Hatch have been answered elsewhere in this closing discussion. The writer disagrees with his first two paragraphs in so far as they apply to high dams on pervious glacial drift; and his fourth paragraph evidently represents his judgment of the action of the type of material encountered at Cobble Mountain Dam near Springfield, Mass. This action is not at all characteristic of water-laid sand.

Mr. Hatch requests a "break-up" of the 30-cent cost per cu yd. It is difficult to sub-divide the actual costs of earth handling into separate items from the daily time sheets, and practically impossible to secure a finer classification of costs than those given in the paper. The over-all earth-placing cost is accurate, however, and as finely sub-classified as need be for the purpose of the paper.

As for the last paragraph of Mr. Hatch's discussion, the writer agrees that "hair-splitting" exactness would require a slight shift of the 100-mm line; but the basic idea demonstrated by Fig. 6 is not affected in any way thereby.

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DISCUSSIONS

ON THE BEHAVIOR OF SIPHONS

Discussion

BY J. C. STEVENS, M. AM. SOC. C. E.

J. C. STEVENS[12], M. AM. SOC. C. E. (by letter)[12a].—The writer's proposal that the coefficient of flow of a siphon be defined as the "ratio of actual flow through the siphon to theoretical flow through its outlet area," has not been challenged. It should be accepted, therefore, by the profession.

The writer suggested further that the discharging efficiency of a siphon be defined as the "ratio of the actual to its theoretically maximum flow," and that the theoretically "maximum flow obtains when one atmosphere is entirely used in producing velocity through the summit section" (neglecting up-stream losses).

This is expressed by Equation (14) and may be easily remembered as the ratio of the velocity through the summit section to the velocity corresponding to one atmosphere. Mr. Nelidov offered a substitute which, if the writer understands correctly, states that the efficiency is equal to the coefficient of flow for the summit section. As the writer pointed out, this is illogical because, if the summit section is constricted, an efficiency in excess of unity is possible. The writer's proposed definition should be accepted.

Mr. Nelidov's criticism of terms deserves mention. "Throat" is not synonymous with "summit." The summit (highest) section of the siphon is meant; not the smallest section. The term, "siphon head," was used to mean the difference in water-surface elevations of head and tail-water. It is not always equivalent to "operating head" which frequently refers to forebay level only. As defined by Professor Etcheverry[7] the "operating head" is measured to the center of the outlet, while "siphon head," as used by the writer, is measured to the water surface at the outlet. "Crest" might have been used instead of "invert," but a distinction was necessary between the upper and lower boundaries of the summit section, and "crown" and "invert" are common terms for these boundaries.

NOTE.—The paper by J. C. Stevens, M. Am. Soc. C. E., was published in August, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1933, by I. M. Nelidov, Assoc. M. Am. Soc. C E.; and March, 1934, by Herbert H. Wheaton, Assoc. M. Am. Soc. C. E.

[12] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[12a] Received by the Secretary March 12, 1934.

[7] "Irrigation Practice and Engineering," by B. A. Etcheverry, M. Am. Soc. C. E., Vol. III, pp. 171-172, 1916.

The writer tried to interest laboratory experimenters in determining losses by laying the model siphon barrel horizontally with its inlet and outlet connected to a head and tail-water tank, respectively. The first to test models of existing siphons in such a manner will learn something about siphon losses. Heretofore, these losses have been considered simply those of inlet, bend, and friction, but the tests cited show that cavitation and, therefore, impact losses must also be taken into consideration. Such impact losses are not subject to analysis by the ordinary model studies, because the velocities to produce them can not be secured. That is what the writer meant by the statement quoted by Mr. Wheaton.

By placing the model siphon horizontally, however, between a pair of tanks where a range of velocities from those in the model to those in the prototype can be secured, cavitation and impact losses can be isolated from those of bends, etc. Professor, E. W. Spannake[13] has been doing some interesting work on cavitation at the laboratory of the Massachusetts Institute of Technology.

Mr. Wheaton attributes the low discharge coefficient, 0.63, of Siphon No. 7 to entrained air, and attempts to show that from his model tests the coefficient should be 0.71 or more if the siphon runs full. As designed, the siphon can never run full. Moreover, there are losses from cavitation and impact, which the writer believes are the real reasons for the differences in the coefficients between the model and the prototype. In the case of Siphons Nos. 5 and 6, entrained air is conceded to have affected the results, giving discharge coefficients of about 0.48, entirely too low; but cavitation also existed.

In Mr. Wheaton's models, air was compressed in their summits as the forebay level, increased, which prevented them from priming except at heads much greater than those corresponding to the prototype. He suggests that the priming hood of the prototype leaked enough to prevent air from being compressed in its summit, but not enough to delay priming. This reasoning does not accord with the facts as later ascertained.

At the writer's suggestion the chief operator of the Leaburg Plant made some experiments to determine the behavior of the siphons as regards priming with the hoods as built with some leakage and also when sealed against air leakage. The lower edges of the priming hoods were not in their lowest position, as was the case when the tests given by the writer were made; hence, the heads necessary to prime are greater than in the writer's tests, but the relative values for sealed and unsealed hoods are very illuminating. The priming gate on Siphon No. 7 was 0.11 ft and that of Siphon No. 6 was 0.19 ft above the invert of crest of the summit section.

For the sealed condition, the priming gate (see "Behavior in Operation" and Fig. 3) was held tightly against its lower guide by iron wedges, and sealing was effected with heavy gear grease. Following are shown the heads (difference, in feet, between siphon crest and forebay water level) to prime:

	As built unsealed	Sealed
Siphon No. 7	0.52	0.30
Siphon No. 6	0.53	0.39

[13] Current Hydraulic Research: Repts. Nos. 1-3 and II-1, U. S. Bureau of Standards (Ref. No. VI-6 I N H-655-C).

If these priming heads are corrected for the distances of the priming gates above the siphon crest (since that depth of water flowed over the crest before the air was shut off) the actual priming heads are found to be, in feet:

	Unsealed	Sealed
Siphon No. 7	0.41	0.19
Siphon No. 6	0.34	0.20

Each of the foregoing tests represents an average of three trials. They show that the sealed siphon primed, roughly, at one-half the head of the unsealed siphon, showing that the air was expelled much faster than it could be compressed by the rising forebay. The leakage around the priming gate increases the time and head of priming instead of reducing it, as the models seemed to indicate. In this regard, the model failed completely to predict the actual behavior of the siphon, and points to another limitation that should be recognized in model work.

Interesting data were obtained during these tests. In order to prevent Siphon No. 7 from priming while Siphon No. 6 was under test, it was necessary to insert air pipes under the hood of No. 7, having an aggregate area of 12.5 sq in., or 2.5% of the area of the smallest section of the siphon barrel.

Mr. Nelidov mentions ice. In 1932 a sharp, cold period brought down some frazil ice and also formed surface ice in the air inlet to the siphons, enough to choke them. The barrels were not affected, but the primers were so choked that the siphonic action would not cease until the water had been drawn below the upper edge of the inlets (see Fig. 3), or 5.25 ft below normal operating level. In order to avoid a repetition of this trouble, holes have been drilled through the concrete to the crowns of the summit sections and 6-in. iron pipe nipples have been cemented into them. These nipples are closed by a screw cap which can be removed to let in air to break the siphonic action whenever the priming hoods become choked with ice.

The writer wishes to commend Mr. Wheaton very highly for the excellent work he has done and the interest he has taken in these siphons. They point the way to the necessity for more intensive work in this regard.

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DISCUSSIONS

STABILITY OF STRAIGHT CONCRETE GRAVITY DAMS

Discussion

By Robert E. Glover, Esq.

Robert E. Glover,[57] Esq. (by letter)[57a].—Under the heading, "Shearing Strength," the author discusses the influence of the stress normal to the shearing plane upon the shearing resistance, and cites tests to show that the shearing resistance of the material is increased by a compression normal to the shearing plane. Punching shear tests made by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and shear tests by S. H. Woodard, M. Am. Soc. C. E., are quoted.

It seems apparent that the purpose of the punching shear tests, and the tests on specimens of Type (*b*), Fig. 2, where the mortar zones were disposed vertically in the specimen, was to permit a direct evaluation of the shearing strength of con-rete when the stress normal to the shearing plane is zero. The conditions necessary for the maintenance of equilibrium require that when a body is subjected to stress in one direction certain intensities of shear and normal stress must exist in all other directions, and since it is known that a pure shear stress is accompanied by compressive and tensile stresses of equal intensity on planes disposed at an angle of 45° with respect to the planes of maximum shear, it becomes pertinent to inquire whether the weakness of concrete in tension would not render such a direct determination impossible.

For a two-dimensional case the question may be attacked as follows: The stress condition in such a solid may be specified completely in terms of the two principal stresses and the inclination of the principal axes to an arbitrary plane of reference. If the material of the body is isotropic and the question

Note.—The paper by D. C. Henny, M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by H. de B. Parsons, M. Am. Soc. C. E.; December, 1933, by Messrs. A. A. Eremin and Calvin V. Davis; January, 1934, by Messrs. William P. Creager, F. W. Hanna, Lars R. Jorgensen, and I. M. Nelidov; February, 1934, by Messrs. Paul Baumann, Thaddeus Merriman, Ivan E. Houk, A. V. Karpov, L. F. Harza, and Edward Godfrey; March, 1934, by Messrs. F. Knapp, and S. H. Woodard; and April, 1934, by Messrs. A. Floris and Joseph Jacobs.

[57] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[57a] Received by the Secretary March 30, 1934.

is one which concerns the causes of failure, only the magnitudes of the principal stresses are significant. It will be found that if there is to be a plane having a zero normal stress, it would be necessary for one of the principal stresses to be compressive and the other tensile. Under these conditions, it can be shown that, if the law of failure by sliding is as stated in Equation (3), the plane of least resistance is independent of the magnitudes of the principal stresses and is identical with that shown in Fig. 1. In order to obtain a shear failure with no stress normal to the plane of shear it would be necessary to make the plane of zero normal stress coincide with the plane of least resistance. It can be shown readily that an experiment made in this manner would probably be terminated by a tension failure before the shearing stresses could be raised high enough to cause a shear failure.

Consider the 1:3:6 concrete described by the author immediately after Table 1, with $k = 0.70$ and $\delta = 62° 30'$. In order to develop a shearing stress of 783 lb per sq in. on the plane of least resistance, with no stress normal to the plane of shear, would require principal stresses of 1 500 lb per sq in. compression, and 407 lb per sq in. tension, respectively. Since the tensile strength of such a concrete would probably be about 300 lb per sq in., it follows that a direct determination with such a concrete may be impossible.

The author records evidence of normal stress in the punching shear tests and notes that the results of the tests on the specimens with the mortar zones "do not agree with the theory of Coulomb." The writer does not believe that the assumption of a uniform shear distribution in the case of the punching shear tests or of the tests on the specimens shown in Fig. 2 (*b*), is warranted, and until the possibility of determining the value of S_1 in Equation (3) by such tests, is demonstrated, will hold to the conclusion that they are inadequate for this purpose. It may be remarked, however, that as long as the designers of gravity dams adhere to the policy of eliminating tensile stresses from their structures, their purposes will be served if the laws of failure within the region where all the stresses are compressive, are determined.

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DISCUSSIONS

ESTIMATING THE ECONOMIC VALUE OF PROPOSED HIGHWAY EXPENDITURES

Discussion

BY THOMAS R. AGG, M. AM. SOC. C. E.

THOMAS R. AGG,[30] M. AM. SOC. C. E. (by letter)[30a].—The discussion of this paper has been gratifying and has brought out some divergent ideas which are of value in clarifying this somewhat illusive subject. Perhaps the writer's ideas as to the place of analyses of this type in highway planning was not entirely clear. It is when the engineer has before him several alternate plans and wishes to determine which is best from the economic standpoint (purely money comparison), that he must undertake some such analysis as that discussed. Frequently the somewhat alluring proposal is wholly without justification upon any utilitarian basis. It may be justifiable on some other grounds but such is not usually the case with the ordinary run of projects of a highway department.

The lack of vital basic data for economic analyses was pointed out by several of those who discussed the paper, and it is to be regretted that so little research is under way in this field. Better information than that now available with reference to highway and vehicle costs, the useful life of roadway surfaces, and related subjects, should be forthcoming. A better classification of highways for the purposes of economic studies would also be of great benefit.

With reference to the use of interest and annuities in building up annual costs for economic comparisons, it is believed that if these factors are neglected wholly misleading results will be obtained. Of course, it is not pretended that interest is paid or that sinking funds are accumulated, but intermittent expenditures of varying sums cannot be placed on a comparative basis without taking account of the time value of money. In principle there is no

NOTE.—The paper by Thomas R. Agg, M. Am. Soc. C. E., was presented at the meeting of the Highway Division, Atlantic City, N. J., October 10, 1932, and published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. W. W. Crosby, Roger L. Morrison, and Samuel B. Folk; December, 1933, by Messrs. W. S. Downs and George E. Martin; January, 1934, by J. T. L. McNew, M. Am. Soc. C. E.; and March, 1934, by H. E. Phelps and C. C. Wiley.

[30] Dean, Div. of Eng., Iowa State Coll., Ames, Iowa.

[30a] Received by the Secretary April 2, 1934.

difference between a public project and a private enterprise in this respect, or at least there should be no difference in the treatment of costs if one wishes a true picture of the relative merits of several possible solutions to a highway problem. If one were planning a financial program for a political unit, the problem would be wholly different.

In correspondence with Professor McNew it has developed that through an inadvertent error, the part of the original paper under the heading, "Grade Reductions," is obviously meaningless. It was condensed from a much longer paper prepared for another purpose and a part of the original paper crept into the revision. In the finished paper, this part will read, as follows:

An exact determination of the economic value of reducing highway grades is impossible because of the heterogeneous character of traffic and the great variation between individual vehicles as to their performance on gradients and the lack of exact information with reference to that performance. However, there is some information available to aid the engineer in forming a correct judgment with reference to the expenditure that may be justifiable for a specific grade reduction project, and such additional data as may be required can be secured by a few simple field observations and at small cost. It is known that little saving in automobile operating costs is effected by reducing those grades that are less than 500 ft long, and that safety to descending traffic is a major consideration when grades are more than 1 000 ft long. In reducing the long grades to a safe rate there is usually a definite saving in operating costs, which can be estimated with some degree of assurance, after certain field data are at hand.

Minor changes in grade may be accomplished by cutting and filling without relocation, but major grade reductions, especially on long hills, are usually accomplished by a combination of relocation and cutting and filling. The changes in grade that may be accomplished by cutting and filling without relocation may or may not have economic value, in many cases they have none.

There are three reasons why excessive grades may increase the cost of highway travel:

First, a round trip on a grade that is beyond the economical limit in rate (whether or not the vehicle must be operated in some gear other than high), results in an increase in fuel consumption over that required for traveling the same distance on the same surface on the level. If a vehicle is operated over various test courses, including level sections and various grades, the ratio of the gasoline required on the grades to that required on the level or on any other grade can be determined. In some studies made at Iowa State College this ratio (designated N herein) varied from about 1.1 to as much as 3.0. It will be taken herein at the lowest value likely to be found for grades ranging between 9 and 12% and the kind of traffic assumed for the illustrative problems that follow. This factor can be determined for specific grade reduction projects with sufficient accuracy by a few days of field work with an assortment of typical vehicles.

The second reason why excessive grades may increase the cost of highway travel is that energy is lost when there is necessity for controlling the vehicle

with the brakes in descending a grade. It would be a simple matter to design a grade upon which a specific vehicle (say, the designer's own automobile) would coast at approximately constant speed or at gradually increasing speed, reaching the end of the gradient at approximately some pre-determined speed; but automobiles of some other design, especially of some other weight, would not perform in exactly the same way. For the greater proportion of the automobiles operating in high gear, with the throttle closed, the safe coasting grade is between 5 and 6 per cent. For commercial vehicles, especially the trailer combinations, the safe coasting grade is between 3 and 4 per cent.

The third reason is that the conventional method of driving is to operate the motor with the clutch engaged when descending a hill and, while thus operating, engine friction and the fuel pumped through the motor both become factors in the problem.

It is apparent, therefore, that highway grades must be a compromise between what is suitable for ascending vehicles and what is safe for descending vehicles. By a series of trials a grade can be fixed that is satisfactory from the operating standpoint and within the limits of the probable savings in the operating costs of the traffic.

The application of these considerations to a specific highway involves only a few rather simple computations and will be illustrated by a numerical problem for which the conditions have been assumed at random.

The physical conditions are set up as illustrated in Fig. 1, which shows a grade of an average rate of 12% and 2 000 ft long, which it is proposed to

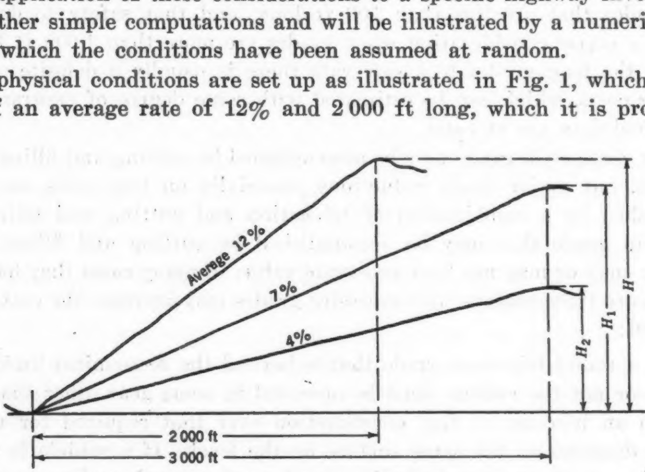

FIG. 1.—ILLUSTRATING ECONOMICS OF GRADE REDUCTION

reduce to 7% by a relocation that requires the new grade to be 3 000 ft long. For the traffic on this road 4% is assumed to be the safe coasting grade; that is, on a 4% grade there would be some vehicles using a little power on the descent and some that would use the brakes, but the braking conditions would not be severe.

For the purpose of these computations it may be assumed that a gallon of gasoline weighs 5.9 lb, tests 19 000 Btu per lb, and costs 19 cents per gal. Each British thermal unit is equivalent to 777.5 ft-lb, and the average efficiency of the automobile and truck engines in use is about 15 per cent.

The quantity of gasoline, in gallons, required to lift 1 ton 1 ft is:

$$G = \frac{2\,000}{19\,000 \times 5.9 \times 777.5 \times 0.15} = 0.00015$$

Assume the traffic on the highway at 4 000 000 tons per year, equally divided between the two directions of travel and consisting of 2 400 000 automobiles of average weight of 1.25 tons, and 250 000 commercial vehicles of average weight of 4 tons. For this problem, ascending grades are analyzed in four steps, as follows:

1.—The annual saving in cost to vehicles ascending the 7% grade as compared to the average 12% grade (neglecting increased distance for the moment) with $N = 1.75$ is,

$$\begin{aligned} S_1 &= 0.19 \times 0.00015 \times 2\,000\,000\ [NH - H_1] \\ &= 57\ [(1.75 \times 240) - 210] = \$11\,970 \end{aligned}$$

2.—Vehicles descending this grade need not use any power and the extra cost of travel for the 1 000 ft of added distance may be neglected since there are no power costs involved.

3.—The computations in Step 1 take no account of the cost of extra distance due to the relocation. There would be no extra power costs for the descending traffic, and the other items may be omitted. The extra costs per year for the ascending traffic will be equal to the cost of traveling 1 000 ft, which is, as follows:

For automobiles:

$$1\,200\,000 \times 0.025 \times \frac{1\,000}{5\,280} = \ldots\ldots\ldots\ldots\ldots\ldots \$5\,680$$

For commercial vehicles:

$$125\,000 \times 0.05 \times \frac{1\,000}{5\,280} = \ldots\ldots\ldots\ldots\ldots\ldots \underline{1\,180}$$

$$\$6\,860$$

4.—Summary for the ascending grades:

Total annual saving by the assumed grade reduction...	\$11 970
Annual cost of extra distance........................	6 860
Net annual saving................................	\$5 110

The conditions to the right of the summit shown in Fig. 1 will be considered in analyzing the descending grades, and, on this basis, the savings will be estimated as were those for the proposed ascending grades. Let it be assumed that for the minus grades, $H = 180$ ft, $H_1 = 150$ ft, the new minus grade is 7%, the old grade averages, 10.5%, and the new grade is secured by a relocation that does not increase the distance of travel on that side of the summit.

The saving per year to vehicles ascending the 7% grade as compared with the average 10.5% grade, with $N = 1.6$, is:

$$\begin{aligned} S_1 &= 0.19 \times 0.00015 \times 2\,000\,000\ [NH - H_1] \\ &= 57\ [(1.6 \times 180) - 150] = \$7\,866 \end{aligned}$$

The total annual saving by the grade reduction is:

On the plus grade..........................	$5 110
On the minus grade.........................	7 866
Total annual saving........................	$12 976

If the investment in grade reduction is to be amortized in 20 years by the annual payment of a sum equal to the estimated savings to present traffic, using an interest rate of 4%, the investment justified in this hypothetical case would be the present worth of an annuity of $12 976 running for 20 years, with interest at 4% compounded annually, which amounts to $176 348.

Similiar computations for a grade of 5% instead of 7%, but with the summit 210 ft high, with distance increases of 2 200 ft and 850 ft on the plus and minus sides of the summit, respectively, will show a probable annual loss of about $1 100.

By way of contrast a similar computation may be made for a plus grade, 700 ft long, at an average rate of 10%, and, consequently, a rise of 70 ft, which is reduced to a 7% grade by cutting at the summit and filling in the sag. The new height will be 49 ft, and there will be no increase in the distance of travel. The factor, N, will be taken at 1.2. The annual saving to vehicles ascending the grade is $1 995.

A new factor now presents itself in that automobiles descending this grade (if the alignment is good) may be permitted to accelerate and thus utilize all the grade effect. The amount that can thus be utilized is the difference between the height of the 7% grade and one of 5% of the same length, amounting to 14 ft. The money value of the height thus conserved is $600 per year. The net annual saving by this grade reduction is, therefore, $1 995—$600, or $1 395. This would justify an investment of about $19 000 on the 20-yr annuity basis of amortization. The value of the grade reduction on the minus side of the summit may be estimated in a similar manner.

In all the foregoing discussion, it is assumed that all the vehicles are driven down grade in gear and with the clutch engaged, as is habitual with most drivers. If "free-wheeling" is used in descending the grades, the foregoing does not apply.

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DISCUSSIONS

THE SURVEYOR AND HIS LEGAL EQUIPMENT

Discussion

By A. H. Holt, M. Am. Soc. C. E.

A. H. Holt,[100] M. Am. Soc. C. E. (by letter)[100a].—The considerable extent of this discussion is taken as a welcome indication of a revival, or at least of the existence, of interest on the part of engineers in the proper establishment of land boundaries. Few of the problems that come to the attention of the civil engineer in general practice possess greater potentialities for eventual importance or far-reaching effect, or call for the use of more mature and experienced judgment, than do those involving the establishment or the recovery of boundaries of land. It is time (as suggested by Mr. Rowe) that the high standards of work required for satisfactory boundary surveys should be recognized, and that work which measures up to those standards be insisted upon.

It is hoped that more States will follow the lead of Massachusetts, and a very few other pioneers, as discussed by Mr. Mueller, to a more scientific method of treating boundary problems. The establishment of a Land Court (placing this class of important, technical problems in hands especially qualified by training—legal and engineering—to rule on such cases), is a long step toward their scientific solution. The registration of title that naturally follows such assurance of competent boundary determination is a matter of great convenience in the purchase and sale of lands.

As suggested by Professor Perry, the local flavor of the paper is obvious, of course. There are two reasons for this: (1) The paper was originally presented before the Iowa Section of the Society; and (2) the apparent necessity of using the law of one jurisdiction as an example, instead of trying to treat the subject more generally, caused the writer to turn naturally to that with which he was most nearly familiar.

Note.—The paper by A. H. Holt, M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1933, by Messrs. Ray H. Skelton, William Bowie, R. Robinson Rowe, and Walter H. Dunlap; December, 1933, by Chester Mueller, Assoc. M. Am. Soc. C. E.; January, 1934, by Messrs. Clarence T. Johnston, A. F. Harley, and Lynn Perry; and February, 1934, by Messrs. C. H. Eiffert and Verne G. Sanders.

[100] Associate Prof., Civ. Eng., State Univ. of Iowa, Iowa City, Iowa.

[100a] Received by the Secretary April 12, 1934.

Quite properly, Professor Johnston emphasizes the need of setting distinctive and enduring monuments. Possibly no other one precaution will tend more directly, or more certainly, to prevent the re-opening of a boundary question than thus to express and mark its solution.

Second only to the importance of suitable monuments is the need of a comprehensive, reliable system of stating their positions, as a means of making possible more definite metes and bounds descriptions, of locating the monuments if they are in place, or of replacing them accurately if they have been destroyed. The obvious answer is the use of such systems of rectangular co-ordinates as are discussed by Major Bowie. In connection with the control surveys recently made in the several States under the auspices of the United States Coast and Geodetic Survey as projects of the Civil Works Administration, plane co-ordinate systems have been established for several of the States. By means of these surveys hundreds of monuments have been set and connected by traverse of second-order precision to the triangulation net. This work has served to make this means of control much more generally available. When the co-ordinates of these monuments have been determined they should, by some means, be made available for general use, and it is hoped that engineers will use them at every opportunity. The advantages of so doing are too well known to require further discussion here. That their use will meet with some opposition when that use involves others than engineers goes without saying. This should not prevent engineers from retaining the courage of their convictions and explaining and promoting their purpose. To attempt to substitute their use abruptly for time-tried methods of stating the positions of property corners would very likely result in encountering such resistance as would defeat the purpose. A metes and bounds description of land can easily be so written that all the advantages of the use of the co-ordinates will be secured, while, at the same time, it would be possible to delete all reference to co-ordinates or to co-ordinate systems and still leave a description that would be entirely acceptable under present standards. This latter feature might make such a description acceptable to abstractors, lawyers, and Courts, in the absence of a formal legalizing act. Such an act might meet with defeat if proposed, thus distinctly hindering the introduction of the use of this control.

The following is suggested as a form of metes-and-bounds description of land, using co-ordinates, referred to the U. S. Coast and Geodetic Survey triangulation, to define the positions of corners:

"A parcel of land situated in Blank County, Iowa, and described as follows. The co-ordinates used to define the positions of corners are referred to the system of co-ordinates established by the United States Coast and Geodetic Survey for use in the southern counties of Iowa. Bearings used are referred to the meridian of that system of co-ordinates:

"'Beginning at a point marked by an iron pin set in concrete, whose co-ordinates are: latitude 571 001.4 feet, longitude 2 461 271.3 feet, and which is on the north line of Section 00, Township 00 North, Range 0 West of the Fifth Principal Meridian, three hundred one and six tenths (301.6) feet east of the northwest corner of said section; thence south seventy-two degrees and forty-seven minutes east (S 72°–47′ E) one thousand eighty-three and four

tenths (1 083.4) feet along a fence line to an iron pin set in concrete, whose co-ordinates are: latitude 570 680.7 feet, longitude, 2 462 306.2 feet; thence south sixteen degrees and twelve minutes west (S 16°- 12′ W) fifteen hundred eighty-nine and three tenths (1 589.3) feet, along and in prolongation of a fence line, to an iron pin, set on the center line of Blank road, whose co-ordinates are: latitude, 569 154.5 feet, longitude 2 461 862.8 feet; thence south sixty-six degrees and five minutes west (S 66°–05′ W) twelve hundred ninety-two and one tenth (1 292.1) feet along the center line of said Blank road to an iron pin whose co-ordinates are: latitude 568 630.7 feet, longitude 2 460 681.6 feet; thence north thirteen degrees and fifty-eight minutes east (N 13°–58′ E), to and along a fence line, twenty-four hundred forty-three and one tenth (2 443.1) feet to the point of beginning; containing forty-eight and thirty-six hundredths (48.36) acres, more or less'."

It is hoped that if attention is drawn to the characteristics of such a description noted herein, it will be possible to show to a client or to a Court the advantages of the use of the co-ordinates, and to indicate the absence of the possible disadvantage of the substitution of something that may seem new for a method that is old and time-tried, although unreliable from the standpoint of the engineer.

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DISCUSSIONS

PHOTO-ELASTIC ANALYSIS OF STRESSES IN COMPOSITE MATERIALS

Discussion

By A. H. Beyer, M. Am. Soc. C. E., and A. G. Solakian, Assoc. M. Am. Soc. C. E.

A. H. Beyer,[12] M. Am. Soc. C. E., and A. G. Solakian,[13] Assoc. M. Am. Soc. C. E. (by letter)[13a].—It was intended that this paper be considered primarily a reconnaissance in the field of photo-elasticity. Some of the possible extensions of this reconnaissance to other studies have been stated very concisely in the discussion by Professor Gilkey; but before any extensive research work can be undertaken along the lines suggested, it is essential that the investigator appreciate fully the practical applications and limitations of this new and interesting method of stress analysis as applied to engineering parts or structures.

Since 1931 a considerable part of the research work at Columbia University has been confined primarily to determining the practical applications and limitations of this method of stress analysis. The writers are convinced that photo-elastic stress analysis, when confined to uni-planar stress distributions in statistically isotropic materials, is reliable, and they have found it to check closely with those stress distributions that can be accurately determined by mathematical analysis.

The method does not lend itself to the analysis of those problems in which ultimate or partial failures occur, as outlined by Professor Gilkey. Photo-elastic methods can only be applied to those stress distributions in which the maximum stresses are well within the elastic limit of the material, because as soon as the elastic limit of the material is exceeded, the stress distribution changes materially.

Note.—The paper by A. H. Beyer, M. Am. Soc. C. E., and A. G. Solakian, Assoc. M. Am. Soc. C. E., was published in September, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1934, by Messrs. H. J. Gilkey and Robert V. Baud.

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[13a] Received by the Secretary April 4, 1934.

As has been suggested by Professor Gilkey, the writers hope that the methods of stress analysis that have been outlined in the paper will be extended in the future to cover some of the more important problems to which attention has been called.

The discussion by Mr. Baud deals largely with the technique of testing and the interpretation of the test results. He suggests that the bands shown in the illustrations should be provided with fringe numbers so that the illustrations may be of more practical value to the reader. This could have been done in the interest of clearness. It is not strictly necessary, however, because, in all important cases for the beam analyzed, the stresses increase as the distance from the neutral axis which, in all cases, has been clearly marked in the illustrations.

The data available lead to the belief that the value for Poisson's ratio is substantially the same for aluminum as for bakelite.

The comparatively large percentage of error in the last column of Table 2 is due largely to the fact that the numerical values for bending moments in the photo-elastic method, on account of the large initial shrinkage stresses present, must be obtained by elimination, an indirect method which is likely to result in small errors. Such a method is practical because the principal stresses for the two conditions are substantially parallel to each other; namely, parallel and perpendicular to the axis of the beam.

The illustrations which show the initial so-called "shrinkage stresses" in the bakelite give the stress distribution only in a qualitative way. This analysis was never intended to be a substitute for quantitative measurements such as could be readily made in a reinforced concrete structural part by obtaining the stresses in the steel from the measured strains in the reinforcing rods.

The paper was not intended to expound in detail the principles of photo-elastic technique and, for that reason, only a few references were given which dealt primarily with the application of photo-elasticity to structural engineering.

Mr. Baud takes exception to the statement in the paper that Fig. 1 is a black and white photograph of the stress fringes taken with circularly polarized white light. In this he is correct; only one ray of the white light spectrum is circularly polarized, the others being elliptically polarized. In taking the photograph, a filter was used to eliminate all except the green light, so as to produce a more nearly perfect circularly polarized light than is possible with white light, without using a filter. Furthermore, the elliptical polarization effect probably introduces no errors of sufficient magnitude to detract from its practical value.

In conclusion, the writers wish to thank the discussers for their valuable criticisms, especially those of Mr. Baud, who (as stated in his discussion) has published some very important papers dealing with both the technical and optical aspects of the use of polarized light in determining the stresses in transparent materials.

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DISCUSSIONS

DURATION CURVES

Discussion

BY MESSRS. DINO TONINI, AND H. ALDEN FOSTER

DR. DINO TONINI[36] (by letter)[36a].—Much information concerning basic concepts that underlie duration curves is brought together in this paper. The application of this type of curve to hydrology is productive of interesting results and its detailed examination always discloses new and elegant characteristics.

As the author properly states it would be desirable that the time unit for original records be in terms of daily flow. Weekly and monthly duration curves can result in appreciable differences especially if the stream flow is regulated only by reservoirs of limited storage. Calculation with daily flow data is almost essential for plotting duration curves of short streams and, in general, for all water-sheds on which the flow is subject to sudden and considerable changes, such as "flashy" streams and those that are fed by melting snow and ice.

The duration curve, plotted by daily flow, in order of magnitude for a record of several years, may represent the "average year." Indeed, the duration curve for the entire record may be considered as the sum of different yearly curves and hence (apart from the time scale) as a duration curve of an "average year." The monthly duration curve, derived as an extension of the average duration curve, seems to have little value in practice. Essential variations in flows between one month and another are too important, having a physically definite character. For instance, it may be desired to know the average distribution of flow in the record of a certain month, say, January; then, the duration curve constructed from all available January records represents an "average January month." Of course, the sum of twelve similar average monthly curves gives an "average generic monthly curve" which is also that of an "average year"; but, as stated, it is inadequate for monthly computations since the distribution of run-off in different months is too vari-

NOTE.—The paper by H. Alden Foster, M. Am. Soc. C. E., was published in October, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1933, by Messrs. Richard Pfaehler, and Edward H. Sargent; January, 1934, by Messrs. C. R. Pettis, and W. G. Hoyt; and April, 1934, by Adolpho Santos, Jr., John D. Watson, Howard L. Cook, C. S. Jarvis, Donald H. Mattern, and F. Knapp.

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[36a] Received by the Secretary January 17, 1934.

able. The duration curve, as an average curve, must be used carefully with records of less than a month; for instance, a duration curve showing the distribution of 24-hr average flows could not represent weekly distribution for glacial rivers, or for large water-sheds on which floods sometimes continue for many days, or for short rivers with sudden variations in flows, etc.

If the storage reservoir is sufficient for 100% regulation, the regulated-flow duration curve is a horizontal line equal to the average discharge of record. On the other hand, the practical development of storage is generally less than that required for complete regulation; hence, the less storage there is available, the more the regulated-flow curve will depart from the straight line of average flow. More important than to provide for a minimum regulated flow, is the problem of securing the best regulation with a given storage reservoir, thus using as much of the natural stream flow above the minimum as is possible with a power plant of limited size. In other words, it is necessary to find, for every available storage, the regulation that permits the maximum utilizaton of natural run-off, with the minimum power plant.

Duration curves and duration-area curves have been used by the Hydrographic Service of Italy for a long time. This Service has also computed all the hydraulic power resources of the Kingdom of Italy, basing the estimates in particular on the application of duration, duration-area, and mass curves. As a result, the officers of the Service, and private engineers as well, have described various mathematical and graphical properties of these curves.

The first application of the duration-area curve seems to have been made in 1920 by a French engineer, A. Coutagne.[37] The Italians, however, took a great interest in the subject, and developed it independently.[38] In Italy, the duration-area curve is called the "hydrologic characteristic of power development."

If q is the run-off and t, its respective duration, the curve, $q = f(t)$, is the duration curve of run-off (generally constructed for data in decreasing order of magnitude (see Fig. 14)). The curve,

$$y = \int_0^q t \, dq \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

is the duration-area curve, plotting values of q on the axis of abscissas and values of the integral Equation (7), on the axis of ordinates (see Fig. 14(b)). The function,

$$y = \int_0^t q \, dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

[37] "Considérations sur les éléments caractéristiques du règime d'un cours d'eau," by A. Coutagne, *Revue Générale d'Electricité*, August 28, 1920.

[38] "I cicli delle portate naturali dell 'Adda Alpina," by Prof. G. Fantoli, *Annali dei Lavori Pubblici*, November, 1926, and September, 1927; "Preliminare esame comparativo delle condizioni idrologiche delle varie regioni italiane," by Prof. G. De Marchi, Relazione del Servizio Idrografico Centrale, Vol. I and III, Memorie e Studi idrografici del S. I.; "Sulla capacità da assegnare ai serbatoi stagionali," by Prof. P. Frosini, *Annali dei Lavori Pubblici*, September, 1928; also, by the same author, "Determinazione della capacità da assegnare ai serbatoi per uso irriguo," *Annali dei Lavori Pubblici*, August 1929; "Su un procedimento statistico applicabile a determinazioni idrologiche," by Prof. L. Gherardelli, *Annali dei Lavori Pubblici*, May, 1929; also by the same author, "Criteri per la regolazione parziale dei bacini imbriferi," *Annali dei Lavori Pubblici*, July, 1929; "Applicazione di metodi statistici all'idrologia," by Prof. Di Ricco, *Bollettino N. 8 dei* Comitato Nazionale Italiano Geodetico e Geofisico, Venezia, 1924.

on the other hand, is a special well-known curve termed a "concentration curve," for which the "ratio of concentration" may be obtained. Theoretically, the latter is equal to zero for a uniform distribution of flow in the period of record, and approaches unity as its upper limit as more of the recorded flow is concentrated in a small part of the entire period.

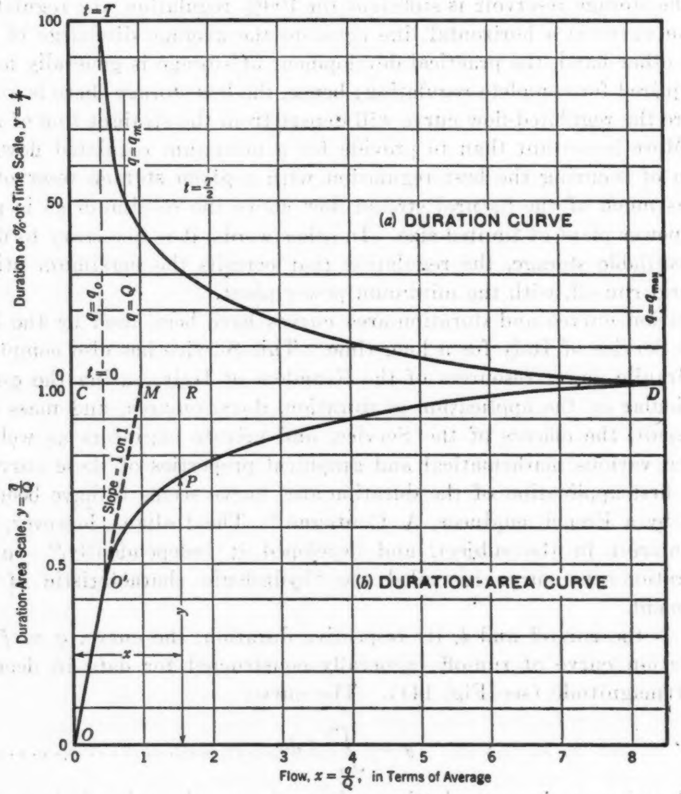

FIG. 14.

Professor Gherardelli[38] has described some new properties of the duration and duration-area curves. He uses the "%-of-time" method for plotting the duration curve, and gives flows in terms of their mean. The abscissas, x, and the ordinates, y, of the duration-area curve may then be defined by the following equations:

$$x = \frac{q}{Q} \qquad \text{(9a)}$$

$$y = \frac{1}{TQ}\int_{o}^{q} t\, dq = \frac{\bar{q}}{Q} \qquad \text{(9b)}$$

in which,

$$\bar{q} = \frac{1}{T}\int_{o}^{q} t\, dq \qquad \text{(9c)}$$

q is the rate of flow; Q, the average or mean flow; t, the duration of the flow; and T, the period of record.

In Fig. 14 (*b*), the duration-area curve which intersects the x-axis at a slope of 1:1, is tangent to the straight line, OM, at O', where $x = \frac{q_0}{Q}$ (q_0 = minimum flow). The curve is also tangent to the horizontal line, CD ($y = 1.0$), parallel to the x-axis, at a point, D, where $x = \frac{q_{max}}{Q}$ (q_{max} = maximum flow). The derivative, $\frac{dy}{dx}$, at any point, x, gives the duration (%-of-time) of the flow, q ($= xQ$); the second derivative gives its frequency.

At a given point, P, of the duration-area curve, the ordinate, y, determines the fraction of average stream flow available with a canal of the maximum capacity, xQ. The segment, PR ($= 1 - y$), represents lost run-off; while the various segments parallel to the axis of ordinates and situated between the curve, OD, and the line, OMD, give the surplus of run-off available with storage reservoirs.

The area, $O'MD$, has an interesting statistical meaning, because it is a function of the relative standard deviation of flow; in fact,

$$\text{Area } O'MD = \text{Area } OCD - \text{Area } OCM$$

From Equations (9),

$$\text{Area } OCD = \int_0^1 x\, dy = \frac{1}{TQ^2} \int_0^{q\,(\max)} q\, t\, dq$$

and from the duration curve, $q = f(t)$ and $dq = f'(t)\, dt$. Hence,

$$\text{Area } OCD = \frac{1}{TQ^2} \int_T^0 q\, f'(t)\, t\, dt = \frac{1}{TQ^2} \int_T^0 \frac{d}{dt}\left(\frac{q^2}{2}\right) t\, dt$$

or,

$$\text{Area } OCD = \frac{1}{TQ^2}\left(\left[\frac{q^2}{2}\, t \right]_T^0 + \int_0^T \frac{q^2}{2}\, dt \right)$$

The expression, $\left[\frac{q^2}{2}\, t \right]_T^0$, is equal to zero, because when $t = T$, $q = 0$; and when $t = 0$, $q = q_{max}$. Therefore,

$$\text{Area } OCD = \frac{1}{TQ^2} \int_0^T \frac{q^2}{2}\, dt$$

Now, since Area $OCM = \frac{1}{2}$, it follows that,

$$\text{Area } O'MD = \frac{1}{TQ^2} \int_0^T \frac{q^2}{2}\, dt - \frac{1}{2} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

The relative standard deviation from the arithmetical average of flows can be computed by the duration curve. The sum of the squares of the deviations is:

$$\int_0^T (q - Q)^2\, dt = \int_0^T q^2\, dt - 2\,Q \int_0^T q\, dt + Q^2 T$$

Since $\int_0^T q\, dt = Q\,T$,

$$\int_0^T (q - Q)^2\, dt = \int_0^T q^2\, dt - Q^2 T$$

Then, letting s equal the relative standard deviation,

$$s^2 = \left(\frac{\text{Standard deviation}}{Q}\right)^2 = \frac{\int_0^T (q - Q)^2\, dt}{Q^2 T}$$

or,

$$s^2 = \frac{1}{TQ^2}\int_0^T q^2\, dt - 1 = 2 \text{ Area } O'MD \text{ ....}(11)$$

From Equations (10) and (11), it follows that Area $O'MD$ is equal to one-half the square of the relative standard deviation from the arithmetical average.

For the duration-area curve Professor Fantoli[38] gives the equation,

$$y = 1 - e^{-r}\text{........................}(12)$$

in which, $r = \frac{x}{n}$, x and y are the same as before, and n is a numerical coefficient. (It varies from 0.77 for the Tevere River to 1.80 for the Arno River.) This coefficient is also represented by Area OCD; for,

$$\text{Area } OCD = \int_0^\infty (1 - y)\, dx = \int_0^\infty e^{-r}\, dx = n \text{ ....}(13)$$

The coefficient is also a function of the relative standard deviation, s. From Equations (10), (11), and (13), it follows that $s = \sqrt{2n - 1}$. Fantoli's formula, Equation (12), is in fairly close agreement with the actual curves.

Coutagne has also considered these matters. For the duration curve he proposed the following:

$$\frac{q - q_0}{Q - q_0} = (N + 1)\left(\frac{T + t}{T}\right)^N \text{ ....}(14)$$

The factor, N, is a new "irregularity coefficient," determined by the flow, $q = q_m$, when $t = \frac{T}{2}$; thus,

$$\frac{q_m - q_0}{Q - q_0} = \frac{N + 1}{2^N} \text{ ....}(15)$$

Theoretically, for a perfectly regular flow, $N = 1$; but in practice it varies between 2 and 5.

The equation for the duration-area curve proposed by Coutagne (using $x = \frac{q}{Q}, y = \frac{\overline{q}}{Q}$) is:

$$y = x - \frac{N\,(x - q_0)^{1+z}}{(N+1)^{1+z}\,(Q - q_0)^z} \quad \text{.................(16)}$$

in which (to simplify typography), $z = \frac{1}{N}$.

The theoretical curve, Equation (12) or Equation (16), must be tangent to the line, $x = y$, at the point, $x = y = q_0$; and, also, tangent to the line, $y = 1$ ($\overline{q} = Q$), at the point, $x = x_{max}$ ($q = q_{max}$). Hence, an approximate duration-area curve can be constructed quickly as a parabola of the second degree fully defined by these points and their tangents (method of tangents).

Another type of empirical duration and duration-area curves is developed from probability curves, as proposed by a French writer, R. Gibrat.[39] Further interesting applications of the duration and duration-area curve are given by a German author, A. Ludin.[40]

H. Alden Foster,[41] M. Am. Soc. C. E. (by letter)[41a].—In the introduction to the paper, the writer indicated that his object was to present a general description of the fundamental properties of duration curves, with the hope that the discussions would bring out additional practical applications. The results have thoroughly justified that hope. The paper has brought out much new information concerning duration curves which, so far as the writer is aware, has not heretofore been published in American engineering literature.

Several of the discussers express regret that the writer apparently had abandoned the use of theoretical frequency and duration curves, and thus appeared somewhat in the light of an apostate. This, however, is far from the truth. The writer is as fully convinced now, as he was in 1924, of the usefulness of theoretical frequency curves for engineering studies. There is no doubt that such methods can be applied advantageously to many types of duration diagrams; but the development of these curves has been rather fully covered by other papers, as has already been noted. In preparing the present paper, the writer purposely avoided any digression from the study of duration curves as actually constructed from observed data. If the fundamental properties of such curves are well understood, the use of theoretical curves should be that much easier.

Two uses of duration curves were indicated in the paper. Colonel Pettis calls these "flood-duration curves" and "power-duration curves." The writer would prefer not to use any such distinctive names, as all the curves are fundamentally of the same type. The difference is only in the manner in which they are used. The theoretical curves are particularly useful for

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[39] "Aménagement hydroélectrique des cours d'eau; Statistique mathématique et calcul des probabilités," by R. Gibrat, *Revue Générale de l'Electricité*, October 15, 1932.

[40] "Bedarf und Dargebot," by A. Ludin, J. Springer, Berlin, 1932.

[41] Asst. to Chf. Engr., Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

[41a] Received by the Secretary April 12, 1934.

probability studies, but if the data can be presented suitably by a smooth curve, there is no reason why the latter should not be used as a picture of what the data would be expected to give if they were more extensive.

The important point is, that a duration curve plotted from the original records, as explained in the paper, can be used to obtain results in actual agreement with those records. If it is desired to obtain results to correspond with what should be expected from a record of infinite length, then the theoretical curve should give more reliable results.

Comparison of Daily and Monthly Flow-Duration Curves.—Mr. Pfaehler, Mr. Hoyt, and Mr. Watson have commented on the error involved by use of a curve based on monthly average flows instead of daily averages. The writer has given considerable attention to this question, which seems worthy of further discussion. The error will always be such as to give results which are too large as compared with what would be obtained from the daily flow-duration curve. An examination of Fig. 6 shows that the area beneath the monthly curve, up to any assumed rate of discharge, is greater than the corresponding area under the daily curve, and this inequality will hold for all rates of flow up to the maximum daily discharge.

A study has been made by the writer to determine the relative amount of this error for a particular stream. Flow-duration curves obtained from the records at three gauging stations on the Delaware River water-shed were used. (The daily flow-duration curves were computed by Mr. E. Laurence Burnett, together with the duration-area values for various rates of stream flow.) The ratio of the daily to the monthly duration-area value was then derived. Fig. 15 with Table 2 shows the variation of this ratio plotted against the stream

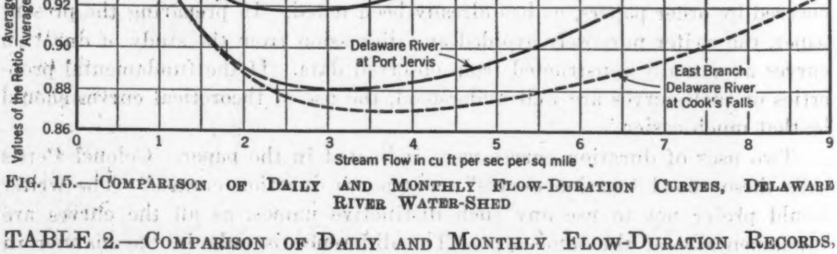

FIG. 15.—COMPARISON OF DAILY AND MONTHLY FLOW-DURATION CURVES, DELAWARE RIVER WATER-SHED

TABLE 2.—COMPARISON OF DAILY AND MONTHLY FLOW-DURATION RECORDS, DELAWARE RIVER WATER-SHED

Location	Period of record	Drainage area, in square miles	Average stream flow from the record, in cubic feet per second	Stream flow, in cubic feet per second per square mile
Trenton, N. J.	1913–1930	6 800	11 500	1.69
Port Jervis, N. Y.	1904–1930	3 070	5 600	1.82
Cook's Falls, N. Y.	1904–1930	241	580	2.40

flow expressed in cubic feet per second per square mile. The curves show that the ratio of the daily to monthly duration-area (or average run-off) value begins at unity for a certain low rate of discharge, which is the minimum daily flow on record. The ratio decreases for higher rates of discharge, and reaches a minimum at about 1½ times the mean stream flow. Beyond that, the ratio increases until it finally reaches unity again at a discharge equal to the maximum daily flow on record.

The three curves in Fig. 15 are for records at points of greatly different drainage area and average stream flow, but they are very similar in shape. It is evident that as the drainage area increases, the ratio values also increase; but the curves for Port Jervis, N. Y., and Cook's Falls, N. Y., are almost identical for flows up to mean run-off, and even the curve for Trenton, N. J., is only 2% or 3% higher than the other two, for rates of discharge less than the mean. Apparently, the curve for Port Jervis could be used without serious error for practically all points on the water-shed farther up stream (except at very high rates of discharge), and also for a considerable portion of the water-shed below Port Jervis.

It is suggested, therefore, that when any extensive study is to be made on a given water-shed, at least one flow-duration curve be computed from a daily record. Satisfactory results should be obtained with a record as short as ten years. The corresponding monthly duration curve should also be computed, for the same period of record. From the resulting duration-area values, the ratios of daily to monthly average run-off are computed, and plotted against the stream flow, in cubic feet per second per square mile. Then, when the record at some other station on the water-shed is to be studied, only the monthly flow-duration curve, and duration-area curve, need be computed The latter can be reduced to an equivalent daily duration-area curve by multiplying by ratios obtained from the curve already plotted. The results should be reasonably precise, even on a large water-shed; and if two daily curves are obtained at points of large and small drainage area, the corresponding ratio curves can be interpolated for intermediate points. The final results should be accurate within a few per cent.

The shape of the ratio curves will doubtless vary for different water-sheds, as the minimum value of the ratio should be smaller for streams which have rapid variation in daily flow (that is, "flashy" streams). The fact that the larger drainage area results in higher ratios is to be expected, since an increase in drainage area has the effect of "dampening" the effect of sudden storms which would cause large daily fluctuations on a small drainage area.

The reason for plotting the stream flow in cubic feet per second per square mile, instead of in terms of mean run-off, is to reduce the effect of variation in drainage area. Since the average flow in cubic feet per second per square mile decreases as the drainage area increases, the ratio curves are brought closer together than they would be if the mean run-off method of plotting were used. However, if only one ratio curve is to be plotted for a given water-shed, either method of plotting would be satisfactory.

The writer wishes to emphasize the importance of making some correction to the monthly flow-duration curve. The present study shows that the resulting error in average flow below a given discharge may be 15%, or more. If it is impossible to construct a daily flow-duration curve for the stream under investigation (due to lack of records, or limited time), a ratio curve derived from the daily records on some other stream could be used as a guide in obtaining a set of correction factors, which would then be applied to the monthly duration curve on the stream in question. The results, even if not very precise, would at least be more conservative than would be obtained from the monthly curves without correction.

Having obtained an equivalent duration-area curve of daily flow, as described previously, it is a simple matter to derive the corresponding flow-duration curve, by means of the slope of the duration-area curve, merely reversing the process explained for the "Duration-Area Curve."

Historical Development.—Several of the discussions present additional information regarding the history of duration curves. Mr. Pfaehler states that he used flow-duration curves in 1908 and 1909. Mr. Sargent gives further reference to his own work on the subject, and shows that a form of duration-area curve was used by him in 1911. Dr. Tonini states that this latter curve was introduced in France in 1920 by Coutagne, and that it has been extensively used in Italy in recent years. Mr. Cook's reference to the paper[30] by the late Mr. Herschel, published in 1878, is of particular interest.

Comments on Discussions.—Mr. Pfaehler has shown how a one-year duration curve may be of value to indicate special conditions in years of drought or excessive flow. He considers the duration curve of regulated flow as not being suitable for use in complicated stream-flow and reservoir studies. This is a question that must be decided by the engineer in any particular case. In fact, as has been mentioned also by Mr. Knapp, each of the graphical methods of studying stream-flow records has its own advantages and limitations. It is only through detailed knowledge of the various methods that an engineer is enabled to select those that are best adapted for use in analyzing a given problem.

Mr. Sargent's statement that computations of power output from duration curves should be discounted is of interest in connection with Mr. Knapp's discussion of this matter.

Colonel Pettis' discussion is chiefly an outline of his methods for determining the probability of floods. It should be of value in bringing his interesting work to the attention of the profession, since this is the first time that his theory has been described in any publication of the Society, so far as the writer is aware. Since the question of floods is very broad in itself, and is somewhat outside the scope of the present paper, the writer will not attempt any further comment.

Mr. Hoyt's discussion is particularly valuable in its explanation of the use of duration curves for international comparisons of river flow and available power. He makes a distinction between "duration curves" and "deficiency

[30] *Transactions*, Am. Soc. C. E., Vol. VII (1878), p. 236.

curves." The writer would prefer to use the term, "duration curve," in a more general sense, so that the deficiency curve would be a special type of duration curve. The writer has always thought of the duration curve as related to its statistical origin. From this point of view, any set of data which can be arranged in a frequency table can be represented by a duration curve; and a monthly or yearly average-flow record can be presented as a duration curve just as logically as a daily or hourly flow record. The essential point in showing such duration curves is, as stated by Mr. Hoyt, that "the explanation must always indicate the time unit used."

Mr. Hoyt states that the duration curves of daily and monthly flow can be used to correct the storage computed from a mass curve based on monthly data. Storage computations by means of the mass curve are generally based on monthly average flows, as the plotting of a mass curve for daily flows is very laborious. The results, however, always involve a certain error, which depends on the distribution of daily flows during the last month of the period of maximum reservoir drawdown. The writer has indicated in the paper that the duration curve by itself does not show the amount of storage actually required. Hence, the difference between the daily and monthly duration curves will not show, theoretically, the proper correction to be applied to storage as computed from the monthly mass curve. However, it is quite probable that the error involved in this method is small, especially since the proper correction to the mass-curve storage value is itself probably within the limits of 10 to 20 per cent. Accordingly, the engineer should be justified in determining this correction for "daily storage"[42] by means of the daily and monthly duration curves, which should give a closely approximate value and should be considerably easier than using a daily mass curve. The writer is indebted to Mr. Hoyt for suggesting this method.

Mr. Santos has given a clear explanation as to how a theoretical duration curve may be computed from the observed data, on which the writer has already commented herein. While a discussion of theoretical curves is outside the limits which the writer intended to set for the paper, reference will be made to Mr. Santos' suggestion whether some other form of curve than Pearson's Type I or Type III might afford a better fit in certain cases. This is doubtless true. It was shown in the paper to which Mr. Santos refers[21] that the Type I and Type III curves of Pearson could be readily computed by means of tabulated factors, using only values for the coefficient of variation and coefficient of skew determined from the recorded data. The method given required no knowledge of the mathematics of skew probability curves. Pearson gave seven types of curves, of which Type II is a special case for Type I, and Type VII is a special case for Type III. Types IV, V, and VI are of a form not commonly met in engineering work, and, moreover, are difficult to reduce to tabular form as was done for Types I and III. It is quite possible that some classes of engineering data might be represented

[42] See "Storage to Be Provided in Impounding Reservoirs," by the late Allen Hazen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 1573.

[21] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

more accurately by these other types; but the computation of the curves is probably outside the ability or patience of most engineers, when tabulated factors are not available.

Mr. Knapp has given an excellent description of the application of the duration-area, or "power-energy," curve to problems of load variation and power output, which requires no additional comment. Of particular significance is his explanation of the use of this curve in determining the output of a hydro-electric plant as affected by the demand requirements. As Mr. Sargent has noted, the duration curve by itself is likely to give results in power output greater than can actually be used. Mr. Knapp would use the theoretical duration curves in preference to observed data, but would treat the duration curve without reference to its related frequency curve. It is interesting to compare his comments in this respect with Mr. Cook's discussion of the properties of frequency curves.

Mr. Watson gives an interesting comparison of the results of computing curves by the "total-period method" and the "calendar-year method." The writer would prefer the total-period method as being more accurate, although it is possible that the second method might be satisfactory in some cases, especially when adjusted by the methods described in the paper to which Mr. Watson refers.[34] His comments on the relative accuracy of daily and monthly flow-duration curves are in close agreement with the writer's conclusions.

Mr. Cook has given a valuable description of the frequency curve, its relation to the duration curve, and its interpretation as regards physical conditions. He notes the difficulty in locating the mode correctly. In this connection, if the Pearson Type I or Type III curve is used, the mode can be located readily by the following formulas:[43] If d = Mean − Mode,

For Type I,

$$d = (cs) \times (cv) \qquad (17)$$

For Type III,

$$d = \frac{(cs) \times (cv)}{2} \qquad (18)$$

Equations (17) and (18) give the distance by which the mode comes below the mean (on the stream-flow axis), expressed in terms of mean stream flow. For the Type III curve, the difference between the mean and the median[44] is approximately equal to $\frac{d}{3}$.

Mr. Cook's remarks on the danger of using too short a time unit in certain cases are significant, and should be compared with Mr. Jarvis' statetments regarding the Upper Mississippi River.

Mr. Mattern does not consider the duration curve suitable for studies of

[34] "An Investigation of the Flow Duration Characteristics of North Carolina Streams," by Thorndike Saville, M. Am. Soc. C. E., and John D. Watson, Jun. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, Fourteenth Annual Meeting, 1933, p. 406.

[43] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), pp. 153, 186.

[44] *Loc. cit.*, p. 157.

power plant operation, in which respect he seems to be in agreement with Mr. Pfaehler. The writer has already commented on this question. Mr. Mattern's suggestion that, because the hydrograph is easier to explain to the layman, it should be used in preference to the duration curve, does not seem fully justified. By the same argument, the engineer should not use calculus in his computations because the president of the company does not understand an integral sign. If the duration curve is helpful to the analysis, there is no reason why it should not be used; if not, then some other method should be adopted in preference.

Dr. Tonini has rendered a valuable service to American engineers by bringing to their attention the work of his compatriots and of other European engineers on the development of duration and duration-area curves. His analysis of the properties of the duration-area curve is particularly valuable. In this connection, it may be noted that the "relative standard deviation from the arithmetical average of flows" is the same as the "coefficient of variation" as used in American practice.

The empirical equations for duration and duration-area curves, quoted by Dr. Tonini, are interesting and may prove of value in some cases. The writer, however, would prefer to avoid the use of such formulas, as he believes greater accuracy will be obtained in most cases by duration curves of the Pearson types, in which the constants are obtained from the observed data in each particular case. The references noted by Dr. Tonini should be useful, particularly the pamphlet by A. Ludin,[40] which contains many interesting suggestions for the graphical analysis of stream-flow data by the hydrograph, mass curve, and duration and duration-area curve.

In conclusion, the writer wishes to express appreciation of the interest shown in this paper, and for the many valuable suggestions and methods which have been made available to the profession by the several discussions.

[40] "Bedarf und Dargebot," by A. Ludin, J. Springer, Berlin, 1932.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ANALYSIS OF UNSYMMETRICAL CONCRETE ARCHES

Discussion

By Charles S. Whitney, M. Am. Soc. C. E.

Charles S. Whitney,[17] M. Am. Soc. C. E. (by letter)[17a].—With the exception of certain unsupported statements which the writer wishes to question, there appears to be nothing in Mr. Molitor's discussion to indicate that the writer's method for the analysis of unsymmetrical arches is not much more simple in application than the orthodox method presented by Mr. Molitor. The latter requires that the rib dimensions be determined completely and that the values of elastic weights and their functions be calculated before the reactions and stresses are determined. The writer's method permits the calculation of reactions and stresses without the necessity of determining first the rib thickness and axis co-ordinates at all points. For this reason, it is possible, by the writer's method, to calculate the stresses in a number of ribs and to arrive at the most economical design in less time than is required for the complete analysis by the orthodox method of one assumed design which, in itself, will not indicate the most economical proportions.

The writer did not begin with a solution for symmetrical arches (as stated by Mr. Molitor), but he did "solve the geometry in the simplest possible manner for the unsymmetrical case." The writer's solution of the symmetrical arch is based on the elastic system used by Mr. Molitor with the rib cut at the abutment only; but for the unsymmetrical case, because of the discontinuity of the arch-axis equation at the crown, it is simpler to cut the rib at the crown, treating the two sides of the rib separately. No difficulties were encountered because of the interruption of the principal system at the crown of the arch.

The writer does not entirely agree with Mr. Molitor that the shape of the arch "should be a three-centered or five-centered curve as nearly as possible coincident with the line of thrust drawn for total dead load plus one-half the equivalent uniform live load over the entire span." As a matter of fact, that

Note.—The paper by Charles S. Whitney, M. Am. Soc. C. E., was published in October, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1934, by David A. Molitor, M. Am. Soc. C. E.

[17] Cons. Engr., Milwaukee, Wis.

[17a] Received by the Secretary April 12, 1934.

method does not have the effect of equalizing positive and negative live load moments. However, if it were desired to use the pressure line for dead load plus a part of the live load, it may be done simply by including the proper live load in the values, w_c and w_s. This is discussed by the writer in his previous paper.[18]

It is not possible, by changing the shape of the arch, to equalize positive and negative moments at all critical sections. Mr. Molitor's statement as to the location of critical stresses at certain definite sections appears somewhat broad, as their location may be affected by load concentrations and the relative importance of stresses due to rib-shortening, temperature changes, etc.

[18] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 966.

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DISCUSSIONS

SOME SOIL PRESSURE TESTS

Discussion

By Messrs. R. L. Vaughn, and M. Hirschthal

R. L. Vaughn,[27] M. Am. Soc. C. E. (by letter)[27a].—One important distinction made in this paper is that ϕ is the angle of internal friction, not necessarily the angle of repose. Other writers have recognized this fact in recent years, but it is one which should be emphasized. The well-known formula for horizontal earth pressure, $P = \frac{1}{2} w h^2 \tan^2 (45° - \frac{1}{2} \phi)$, was evolved long ago. The derivation was for an ideal granular material presenting no other internal resistance than that due to friction. For such a material it could be demonstrated that the angle of internal friction was equal to the angle of repose.

At a somewhat later date the proposition was demonstrated that when such an ideal material was submerged the same formula for horizontal earth pressure would apply, providing that, in such case, w be taken as the weight of material when submerged, in addition to which there would be full hydrostatic pressure corresponding to the given head of water. The present series of experiments marks the first attempt, of which the writer has any knowledge, to verify the last conclusion. The question is of importance in connection with the design of bulkheads and quay walls to retain submerged fills.

The author refers to ϕ as the angle of internal friction. Exception might be taken to this definition, and it is suggested that a better term would be "angle of internal resistance." It would be difficult to conceive of any manner in which the internal friction in compacted sand could be markedly increased when the sand was saturated, and it is evident that some property other than friction is brought into play by the inundation. This additional property is generally referred to as "cohesion," and it is supposed to be due to capillary action of the water in the small passages between the grains of sand. If this is the true explanation, then one might expect to find a much greater increase in internal resistance when sand was saturated than when

Note.—The paper by H. de B. Parsons, M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1934, by J. C. Meem, M. Am. Soc. C. E.; February, 1934, by Messrs. Eugene E. Halmos, and L. C. Wilcoxen; March, 1934, by Messrs. O. K. Froehlich, H. L. Thackwell, and Jacob Feld; and April, 1934, by Messrs. T. Farrance Davey, D. P. Krynine, and Charles Terzaghi.

[27] Cons. Engr., San Francisco, Calif.

[27a] Received by the Secretary March 21, 1934.

gravel was saturated because the water passages in the latter are much larger. The results of the author's tests tend to confirm this explanation.

If it is granted that the angle of resistance is due to a combination of friction and cohesion, it follows at once that the value of ϕ will vary with the depth of material, and the mathematics of the investigation become quite complicated. In addition, there is no experimental proof, as far as the writer knows, that either the friction or the cohesion remains constant with a varying water pressure. For instance, if the bank of sand 7 ft high, the pressure of which was measured by the author, had been resting against the base of a dam 200 ft high the results might have been far different. It scarcely seems likely that such would have been the case, but the contrary has never been proved.

While in no wise wishing to disparage the author's results, which are valuable and interesting, the writer desires to point out that the tests are on a bank of sand only 7 ft high. They show a most marked change in the coefficient of internal resistance, for sand, upon inundation. This may be caused by the development of additional internal resistance due to capillary action. The numerical value of such capillary resistance, presumably, is limited. If this is true, its effect upon the coefficient of internal resistance would decrease as the total of such internal resistance, due to all causes, increased; that is, the additional resistance due to cohesion in the test case caused a change of 100% in the coefficient of internal resistance; but if this same amount of cohesion had been introduced into a bank of sand 100 ft high in which the true internal friction was many times as great, then upon inundation the change in the coefficient of resistance might have been relatively insignificant.

Actual experimental proof is urgently needed as to whether this relative change in the value of tan ϕ upon inundation holds good for high banks of soil, or whether as the depth becomes greater the importance of the observed effect decreases. Of even greater importance is the need for exhaustive experiments regarding the behavior of the cohesion in finer grained soils, such as silts and clays, under heavy pressures.

These questions are anything but academic. There is at present in prospect a resumption of hydraulic mining in California. Existing laws make mandatory the construction of dams to retain the débris from such operations, in order to avoid the silting up of the main rivers. There is a wide divergence of opinion among engineers as to the basis upon which such dams should be designed. One school claims that the pressure against such dams will actually be less than that due to a full head of water. Another group of engineers, in a position of some authority, insists that such dams shall be designed to withstand a pressure due to a liquid having a weight of 160 lb. per cu ft. In many cases, if not all, the decision as to whether or not mining shall be resumed will depend upon the cost of the necessary débris dam. Some of these dams may be considerably higher than 100 ft. No actual information concerning earth pressures, whether or not of saturated material, involving heights even approaching these, is known.

Direct measurements of ϕ by the method described in the paper, or by any other method, are difficult to make with any degree of accuracy. The fact that in this instance such direct measurements gave higher results than those derived from the observed pressures suggests the possibility that cohesion was present in the material when the direct measurements were taken. The effect of such cohesion would be to increase the value of tan ϕ for light loadings. Had the upper box in the measuring apparatus been weighted to give the average unit pressure that existed in the bin, the agreement between observed and derived values might have been closer.

It does not seem logical to suppose that the loss in total pressure upon partial inundation could be due to loss of weight in the submerged material, as the author suggests. It is true that when submerged the material loses weight, and that, if the value of ϕ were constant at all times, the pressure from the submerged earth would be less than that from the same earth when not submerged. However, the water of submergence exerts an additional pressure and, upon the principle that action and reaction must be equal, it would be impossible for the water, through its buoyant action, to take away more pressure than it itself added through its presence.

It is suggested that a more plausible explanation of this unexpected phenomenon may be found in the varying value of ϕ. The total pressure against the bulkhead may be regarded as made up of four parts: (*a*) The pressure of the dry earth, above the line of submergence, against that part of the bulkhead with which it comes in contact; (*b*) the pressure of the submerged earth, due to its own weight only, against that part of the bulkhead with which it comes in contact; (*c*) the pressure of the water against the submerged part of the bulkhead; and (*d*) the pressure against the submerged part of the bulkhead, exerted through the submerged earth, due to the load or surcharge upon that earth, caused by the dry earth above. The weight of this surcharge would not be the weight of the earth above, but would be that weight decreased by the amount of the friction of such earth against the face of the bulkhead, and the distribution of this imaginary surcharge would vary with the distance from the bulkhead. In computing Part (*a*) w and ϕ for dry sand would be used; for Part (*b*) the corresponding values for the inundated condition would be used; while for Part (*d*) the amount of the surcharge and its distribution would require further investigation and the value of ϕ for the submerged condition would be used. To put the idea in other words: When some of the material is submerged that portion loses weight, and there is a consequent loss in pressure. Furthermore, since ϕ is increased, the pressure due to the weight is further decreased; and, again, the weight of the unsubmerged earth above causes an additional pressure to be transmitted to the bulkhead through the submerged earth; but if the value of ϕ is increased, then the pressure so transmitted will be decreased. On the other hand, the water of submergence itself exerts a pressure against the bulkhead. Until some considerable depth of submergence is reached, the sum of the three losses may exceed the gain due to the presence of the water.

The suggestion that ϕ may show a marked and hitherto unsuspected increase on submersion, at least in the case of sands, is most interesting. If this result should be confirmed by further experiments and for greater depths, it might help to explain several observed facts which have not been accounted for in a completely satisfactory manner. Hydraulic dredge operators have long known that an otherwise deficient bulkhead could be relieved by pumping an embankment against it before completing the remainder of the fill. Just why this should be so, particularly where the material was a permeable one, such as a sand, has not been entirely clear, since before the fill could be completed the water level must be raised to the top, apparently giving a full hydrostatic head against the bulkhead in any event. The most common idea has been that the embankment first deposited had time to settle, consolidate, and develop cohesion. Undoubtedly, these factors had some effect, but the explanation has never been completely satisfactory to the writer and he has been inclined to suspect that leakage through the bulkhead, together with the well-known loss of head experienced by water flowing through earth, materially reduced the hydraulic head effective against the bulkhead. In other words, after the bulkhead had been protected by some width of earth the cracks and leaks in it served the same purpose as the weep-holes that every one knows should be placed in all retaining walls. The author's results indicate that, particularly in the case of fills which were partly submerged, an unexpectedly high value for ϕ might furnish a partial explanation. However, such a consideration could not explain how some bulkheads which evidently could not withstand a full water head alone, not to mention an additional load due to earth, could retain a dredger fill, and the writer still thinks that loss of head due to leakage is an important factor. It would be interesting to run another set of tests with the author's apparatus, leaving a number of small leaks in the bulkhead.

Most bulkhead design is an empirical affair at best. Some modern types of steel sheet-pile bulkhead are practically water-tight. If a steel bulkhead were designed so that each part of it were mechanically equal in strength to the corresponding part of some wooden bulkhead which experience had shown to be adequate, the designer might get into trouble because there was no leakage to relieve the water pressure.

Bulkheads depend on the passive resistance of the earth outside their toe for stability. Through a process of reasoning similar to that by which the pressure formula was derived, it can be shown that this resistance (sometimes called "passive pressure") is equal to: $R = \frac{1}{2}\, w\, h^2 \tan^2 (45° + \frac{1}{2}\,\phi)$. Both these formulas are not even approximately true for cases in which cohesion plays an important part, unless the pressure is measured first and then ϕ is computed to suit; in other words, if the answer is known, the formula can be "patched up." Any values of ϕ obtained by taking published "angles of repose," or by measurement of surfaces of repose, or by direct measurement of the same material under different conditions of moisture content and unit pressure, will give seriously erroneous results when substituted in either of these formulas, if cohesion is present at any time.

A number of reliable European experimental investigators have reported that the passive resistance was at least twice that given by the formula, for any sand or gravel having an angle of repose not flatter than two horizontal to one vertical. No one has ever proved that there was any inherent relation between the angle of repose and the angle of internal resistance (which governs pressures), except in some ideal case that probably never would be encountered in Nature. Furthermore, there was nothing in the accepted theory to account for this added resistance. It has seemed dangerous to rely upon such a factor in preparing a design, at least until some explanation was advanced which would indicate that the effect would be present invariably. The writer has been inclined to believe that the effect was reliable, and was due to friction on the face of the bulkhead, which would tend to increase the effective weight of the material that furnishes the passive resistance. However, in some cases at least, the effect could be explained through unsuspected variations in the value of ϕ which the author's tests indicate may be more common than might be supposed.

Comment is made in the paper concerning the slope assumed in Nature by banks of the material tested. This is by no means a rare phenomenon. It is a matter of every-day occurrence in the dredging business, in which case the sides of channels generally assume some kind of a curve, concave upward, and the upper part of the curve may approach the vertical. Such slopes bear scant resemblance to the natural side slopes not flatter than (say) two to one frequently mentioned in the specifications, and shown by straight inclined lines on the cross-sections. There have been many arguments about side slopes between dredging contractors and engineers. These curved slopes are doubtless due to the effect of cohesion or some similar property; theoretically a true granular, cohesionless material should assume a straight slope, but few materials ever do so under water.

M. HIRSCHTHAL,[28] M. AM. SOC. C. E. (by letter)[28a].—The difficulties encountered when making earth-pressure tests to obtain results that simulate actual conditions in the field are suggested in this paper describing Mr. Parsons' painstaking tests. For example, there is an utter lack of test data as a basis for studying the action or variation of earth pressure with increasing heights of fill and for different types of surcharge on such fills.

From the standpoint of the railroad engineer this particular problem is of great importance since a wide variation of conditions is encountered in railroad embankments. Observations on structures built in connection with such embankments have shown ample evidence of varying angles of repose in the same material for varying heights of embankment. Furthermore, examination of earlier structures built on the Delaware, Lackawanna and Western Railroad clearly indicates a flattening of the slopes with increase in heights of embankment such that the slopes in Table 9 have been adopted for use in the design of retaining walls of various heights, and in the determination of the dimensions of structures supporting such embankment.

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[28] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

[28a] Received by the Secretary April 4, 1934.

TABLE 9.—DESIGN DATA FOR EARTH PRESSURES

Height, in feet	$\tan^2 \frac{1}{2} (90 - \phi)$	Angle of repose	Slope
Up to 20........	0.286	33° 41′	1.0 on 1.5
20 to 30........	0.300	32° 33′	1.0 on 1.57
30 to 40........	0.315	31° 24′	1.0 on 1.64
More than 40....	0.333	30° 00′	1.0 on 1.732

It was found that even with this flattening of slope the fill would "slop" over the wing-walls, or over the parapets of supporting arch barrels, so that present practice is to make provision for a slope of 1.0 on 1.7 for all embankments between 20 ft and 40 ft, increasing the allowance for heights greater than 40 ft.

These results are bound up with the effect of sloping surcharge. The effects of both uniform and sloping surcharge have not been sufficiently analyzed and determined. It is quite certain that a surcharge of both types produces a change in the angle of "internal friction" as well as in the angle of repose of the underlying material in the course of the compaction of this material.

Mr. Parsons calls attention to the fact that the Rankine or Coulomb theory, for a fill without surcharge, yielded varying results for the angles between the horizontal and the planes of internal friction for dry soils and fully saturated soils. As far back as 1912 the writer emphasized[29] the inconsistency of results obtained by the application of either the Rankine or the Coulomb theory of earth pressures to uniform, as compared with sloping, surcharge for certain conditions.

The only reliable results obtainable as a guide for future design are to be sought (and found) through the installation of pressure-cells in actual construction, so as to register the pressure of the filling material against the retaining walls and abutments for the various conditions, including that of a surcharge that may obtain at a definite site and the resulting soil pressure on the foundations.

Engineers entrusted with such design and construction are earnestly urged to induce their clients to install these cells, in order to obtain the very valuable resulting information as a guide for subsequent design.

The question of weight of material, of course, is an interesting and important one in connection with the pressures against the structure, and no doubt the various degrees of saturation modify the results. There is an anomalous condition, however, produced by saturation, inasmuch as complete saturation may produce pressures apparently greater than hydrostatic pressures due to the greater apparent weight of the filling material than that of water; whereas the water produces an upward pressume causing buoyancy and a reduction in the weight of the filling material. This compensating effect of buoyancy would naturally reduce the resulting horizontal pressure and

[29] *Engineering News*, April 25, 1912, p. 799.

under certain conditions, possibly, bring it within the range of the pressure exerted by the dry material. These results will be greatly modified, of course, by the grading of the filling material.

The weight of the filling material is of secondary importance in the design of retaining structures since it also plays the part of aiding the stability of the structure by means of the vertical component of the resultant pressure where the design involves a base or footing projecting into the fill, a batter in the direction of the fill, or a sloping surcharge.

Retaining walls never fail in shear through any section, and infrequently by sliding on the base. Most failures are due to excess of pressures at the toe (or abutment) resulting in possible overturning. A slight difference in the location of the resultant at the base will produce a marked difference in the maximum soil pressure at the toe, and this difference in the location of the resultant is traceable to the assumption of the angle of "internal friction" (or angle of repose) of the filling material in its action causing the overturning moment due to the horizontal thrust.

The question of internal friction is another puzzling one when it is considered that the filling material may be a uniform sand, each granule being an independent little mass more or less globular; or, it may be a rock fill laid in layers, with dirt filling the voids.

All these points indicate the need of further research on this important subject of earth or soil pressures—including foundation pressures, which are really passive resistance of the soil to superimposed pressures.

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DISCUSSIONS

MODIFYING THE PHYSIOGRAPHICAL BALANCE BY CONSERVATION MEASURES

Discussion

BY MESSRS. W. P. ROWE, AND J. C. STEVENS

W. P. ROWE,[42] ASSOC. M. AM. SOC. C. E. (by letter)[42a].—The subject of this paper is timely for Southern California, because of the extensive flood-control and water-conservation works being built under the Federal Unemployment Relief Administration in this area. The author sub-divides the effect on the physiographical balance of conservation measures, under three headings: (*a*) Changes in the water-shed cover; (*b*) effect of regulation on the natural balance of a stream system; and (*c*) effect of débris barriers on the stability of the stream bed on the débris cone. He assumes that these three effects are the work of Man. The water-shed cover is the work of Nature, and it is axiomatic that Nature is never static. Climatic conditions are a governing factor in water-shed cover and one over which Man has no control.

The disaster in the La Crescenta-Montrose area of Southern California (New Year's Day, 1934), resulting from an intense rain (averaging more than 12 in. for the 2-day storm, and with a fairly uniform maximum intensity of about 1 in. per hr for 3 hr over the entire area), on a small water-shed which had been recently denuded by fire, has concentrated attention on water-shed cover. All the measures for fire prevention, advocated by the Forest Service in dealing with brush-covered areas, such as paved highways, motor-ways, fire trails, fire breaks, fire apparatus, and hundreds of men from near-by unemployment relief camps were available, and still the fire burned unchecked until the area was burned over. The fire was held at the crest of the drainage area by back-firing, which is nothing more nor less than controlled burning. When a torrential rain fell on this area, the catastrophe resulted. The natural phenomena accompanying this disaster offer the clearest examples as to how the many enormous débris fans have been formed at the mouths of Southern California streams. The occurrence of these fans is evidence of similar but heavier floods in the past.

NOTE.—The paper by A. L. Sonderegger, M. Am. Soc. C. E., was published in December, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1934, by Messrs. H. H. Chapman, and E. B. Debler; and April, 1934, by Messrs. Frank E. Bonner, and C. S. Jarvis.

[42] Cons. Engr., San Bernardino, Calif.

[42a] Received by the Secretary February 28, 1934.

Segregating proposals for improving the water-shed cover Mr. Sonderegger, in Article 2, suggests two classes. Regarding the first of these (as it relates to the capacity of the water-shed cover to "induce more copious precipitation"), the rain in the La Crescenta-Montrose area, where there was apparently as much rainfall on deforested areas as on those which were heavily forested with brush, would seem to disprove this theory; at least, as regards local conditions in Southern California.

In treating the second class it should be emphasized that reduction of evapo-transpiration loss is not the primary reason for the advocation of light deforestation or controlled burning. The fire in the La Crescenta-Montrose area demonstrates the advisability of some method of light deforestation in the brush-covered areas, which would permit no more than 50% of the water-shed cover to be denuded at any one time. Where the entire water-shed cover was destroyed, the resulting damage was heavy, but in the adjacent areas, where less than 50% of the water-shed area was burned over, there was little material damage. The fact that this area was denuded, in the face of all the preventive measures recommended and used by the Forest Service, would indicate that the present policy is wrong. If light deforestation were advocated simply as a means of decreasing evapo-transpiration losses and thereby increasing the water supply, the present Forest Service policy of fostering a fire hazard, which eventually results in the temporary destruction of the entire water-shed cover when the inevitable fire occurs, offers the best medium. The inevitable needs no advocate.

That the effect of ashes is to seal the stream bed, is a moot question. The presence of ashes in an absorptive stream bed, after a forest fire, is an indication that the water has percolated through and left the ashes behind. Any flood will stir up the stream bed and carry away the transportable material, the size of which will depend on the severity of the flood. When the peak of a flood has passed, lighter and still lighter materials will be dropped as the flow decreases, until, eventually, only the finer materials are left to be deposited when the water sinks. The argument relating to ashes has been used in the past as regards the presence of silt in a stream bed after a flood. The present methods of maintaining silt-suspending velocities in the spreading ditches have solved this problem. It is significant that no large deposits of ashes or silt have ever been found, as far as the writer knows, in excavations made in active stream channels on Southern California débris cones.

In connection with the acts of Man on the effects of over-balancing the regimen of the stream, Mr. Sonderegger has pointed out the effects of the deposition of débris resulting from the construction of dams or barriers across the channels. The Government's activities for unemployment relief in Southern California have resulted in large sums of money being spent on water conservation works on the débris cones at the mouths of Southern California streams. Dams have been built across them without any regard for the future effects of deposition of additional débris.

In 1926, a spreading dam, 2 000 ft long, was constructed across the mouth of Lytle Creek. This dam raised the bed of the active channel at least 10 ft.

In succeeding years, floods re-graded the stream to such an extent that it has been raised at least 2 ft at the U. S. Geological Survey gauging station 1.5 miles up stream. The future effect of this dam would have been a shifting of the stream to a highly improved area on its débris cone west of the old active channel, if an expensive training levee confining the stream to its former active channel, had not been constructed. This reconstruction resulted in the abandonment of about 1 750 ft of the original dam.

In 1920, a spreading dam, 3 000 ft long, was constructed on the débris cone of the Whitewater River. This dam was built on an approximate contour to a height of 5 ft, but with the maximum height limited to 3.5 ft above any of the active channels traversing the cone. The other 1.5 ft of height was buried. Floods which followed the construction of the dam resulted in the re-grading of the stream beds up stream, until the water was forced out of its former channels and spread over an area which the stream had not occupied for at least twenty years. Precise profile lines run across the cone from this dam in 1920, 1924, and 1927, show that each flood gradually raised the bed up stream until, after the capital flood of 1927, the re-grading effect was noticed at least 1 mile up stream. This was one of the primary reasons for the construction of this dam, as no danger could result from such additional overflow of adjacent lands.

A careful survey of the débris cone was made in 1927 by running an axial line down the center of the cone and parallel radial lines on the arcs of circles having their apexes at a common point where the stream first begins to fan out. This survey showed that there was a true cone between the confining banks of the wash. Future dams are planned to take advantage of this condition by installing movable crests so that the stream bed may be raised or scoured at will. In addition to the re-grading effect up stream, there was a scour down stream due to the erosion of the 1.5 ft of material between the original stream bed and the spillway mat 5 ft below the crest of the dam. This scouring effect extended down stream for a distance of 700 ft.

The author has shown correctly that the effect of the construction of bridges, culverts, dikes, and roadway dips across débris-laden streams is not a simple matter, but one that should receive careful study, so that the various structures along the same stream shall be co-ordinated in their effect on the deposition and scouring of the débris.

Another factor in the deposition from débris-laden streams is the elimination of artesian areas through which such streams may flow. In the San Bernardino Basin, the artesian rim in 1916 extended to a point 5 miles, or more, above the Bunker Hill dike, and no deposition took place in this stretch of the stream while the artesian rim remained at this limit. However, with increasing use of water and years of sub-normal stream flow, the artesian area shrank until, in 1934, there are more than 3 miles of absorptive channel where there was formerly a rising stream. This has resulted in deposition of débris by the diminution of the stream flow due to absorption. At the Gage Canal intake, 2 miles above the Bunker Hill dike, there is a sand-

trap in the canal which formerly drained water and sand back to the stream level. One can now stand in this sand-trap, and his head will be below the level of the present stream channel.

In designing barriers across such streams, the effect of scouring during flood periods has not been considered in many cases. The writer has found tin cans and other signs of recent origin at a depth of 5 ft below the active stream bed, indicating scouring to at least this depth during flood. A sawed plank was found during the excavation for Boulder Dam at a depth of 50 ft below the present bed of the Colorado River.[43] Stream gaugings at Yuma, Ariz., indicate that the Colorado River scours to a depth of more than 30 ft during capital floods.[44] When a permanent barrier is placed across such a stream, the flow that was formerly transported through a scoured section is forced to pass over the barrier, thus increasing the wetted area above that barrier. In many cases the only way in which this increased flow can be accommodated is by overflowing adjacent lands which were formerly in no danger from floods.

Another active phenomenon that influences the physiographical balance of streams in Southern California is the effect of changing climatic conditions. The Whitewater River flows into the Salton Sea, which in recent years (but not within the memory of Man), extended to an elevation 40 ft above sea level. The present level of the Salton Sea is about 250 ft below sea level. While it was at its former level of + 40 ft, the stream bed of the Whitewater River was graded to fit this condition; since the lowering of the sea level, the stream is accommodating itself to the changed condition by scouring its bed. This scouring is an almost imperceptible process because of the long intervals between capital floods. Whenever such a capital flood occurs the erosion proceeds up stream as a series of cataracts.

In former years the Mojave River adapted its bed to fit the conditions created by the ancient Manix Lake into which it flowed. This lake had a surface area large enough to evaporate more than five times the present mean annual flow of the river. After the barrier that formed one side of the lake was washed away, the water in the lake escaped and scoured a deep outlet. The stream is still adapting its grade to this new condition, and in this case, also, it is slow because of the infrequency of capital floods.

The physiographical balance on débris cones is a delicate one. There is ample proof that slow climatic changes (imperceptible from year to year, but apparent over long periods of time) are modifying this balance. Annual precipitation data are misleading. A year of well-distributed rainfall will move a certain quantity of débris and will deposit it as the water sinks into the cone. Another year of equal rainfall, but torrential in character for a few days or hours, will have a far more devastating effect.

Professor Ellsworth Huntington has given a vivid description of alluvial terraces as proof of climatic changes.[45] Similar evidences of change are to be

[43] *Engineering News-Record*, December 21, 1933, p. 762.

[44] "The Colorado River Problem," by William Kelly, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 322.

[45] "The Climatic Factor," by Ellsworth Huntington, *Publication No. 192*, Carnegie Institution, pp. 23 to 36.

noted on the various coastal streams of Southern California. Changes of stream-channel conditions, that can not be attributed to over-grazing, forest fires, or any act of Man have occurred within the memory of man.

The New Year's Day, 1934, flood in Pickens Canyon in the La Crescenta-Montrose area exposed evidence of similar floods and canyon-side slumping in previous years. At one place, a log about 12 in. in diameter, that had been buried long enough to rot through, is exposed. At another place, a flood had carried down an oak tree and buried it in a horizontal position. Its roots were buried and several sprouts came up from the trunk. These sprouts had developed into trees as large as 4 in. in diameter before the last flood exposed their origin. The fact that some agency was instrumental in building more than a hundred check dams in this narrow bed-rock canyon would also indicate that some one with previous experience had recognized the danger from débris or mud flows.

The carrying capacity of a stream loaded with sand, gravel, and boulders may be calculated, but when a large quantity of mud increases the specific gravity of the conveying fluid, the carrying capacity is tremendously increased. A flood of clear water of sufficient force to move some of the boulders found on the local débris cones is inconceivable, but when there is a flow of mud accompanying the flood one can easily account for the presence of these boulders.

Mr. Sonderegger has mentioned débris barriers above the canyon mouths as a means for preventing the débris from reaching the cone. The Haines Canyon débris basin, which was credited with such miraculous powers during the La Crescenta-Montrose disaster, was simply a gravel-pit in the stream channel. Its lower end was closed with the waste sand from the screening plant. The confining wall had been lined with wire and rock protection and equipped with a shallow flat spillway lined with concrete. All the storage capacity was below the original stream bed at the outlet end. The excavation of rock and gravel at the upper end of the pit had created a drop of more than 30 ft at the entrance of the stream channel. When the flood occurred, the water that flowed over this loose alluvial bank soon cut a new grade and deposited the eroded material in the basin. This recession of grade worked up stream through several wire and rock check dams and was stopped just below the gauging station about ½ mile up stream. At this point the grade was lowered about 5 ft. The material eroded in this section of the stream accounted for most of the débris deposited in the basin. Had the barrier been erected above the stream bed, most of this scouring would not have occurred, and there would have been more storage space for débris from the upper water-shed. In such a case, the dam and spillway would have to be of the safest and most permanent construction. It would probably be cheaper to re-excavate the débris after each flood than to construct such expensive works. This would depend on the frequency of débris-carrying floods.

The high peak flow of 4 000 or 5 000 sec-ft per sq mile credited to Haines Canyon[46] was undoubtedly caused by log jams in the canyon backing up the

[46] "Notes on the Recent Flood," by E. C. Eaton, M. Am. Soc. C. E., (Not published.)

water and suddenly giving way. That this flow was of short duration is proved by the outflow through the spillway of the débris basin. The area of the Haines Canyon water-shed above the débris basin is approximately 1.2 sq miles, about one-half of which was burned over. At the critical section of the spillway, the average depth of water was 2.2 ft and the average width was 45 ft. The concrete approach channel is level, but just below the critical section at the crest there is a drop of about 1 ft in 10 ft. Computations based on these data indicate that the outflow did not exceed 400 sec-ft.

The area of the Haines Canyon débris basin below the spillway level was not greater than 2.5 acres before the flood. During the flood a débris cone was built at the upper end of the basin by inflowing débris. The present area (1934) at the high-water mark is 1.9 acres. According to an eye witness, the water level in the basin was 3.5 ft below the spillway level at the time the peak flow arrived, so that there was a capacity of about 9 acre-ft for storage before overflow began. The float-well at the U. S. Geological Survey gauging station silted up during the peak flow. A total of 7 acre-ft passed the gauging station during the three days preceding the peak. This value, less seepage losses, is an indication of the original storage below the aforementioned 3.5-ft mark. Between the spillway level and the high-water mark there would have been about 5 acre-ft of storage while the peak outflow was occurring. If the inflow was assumed to be 4 000 sec-ft for only 3 min. and the outflow was 400 sec-ft for the same period, the available storage would balance the flows.

The same witness stated that the outflow continued for 2.5 hr. The inflow during this period could not have exceeded the outflow as there was no available storage for regulation. If all the rain, falling on a burned-over area of 300 acres, at a rate of 1 in. per hr, became available as run-off and one-fourth of the same amount of rain falling on 400 acres of brush-covered area joined it, one would have a run-off figure consistent with the peak outflow as computed previously herein. It is evident that the high peaks were caused by the sudden release of dammed water and that steps should be taken to prevent the recurrence of such a condition in the future.

The approach to the spillway of the Haines Canyon débris basin is crossed by a railroad trestle for the operation of the gravel pit. The trees carried down by the flood partly blocked the approach and it is conceivable that the outlet might have been blocked completely by trees and other drift, in which case the débris barrier would have failed, resulting in an enormous damage. The elimination of similar hazards to the proposed small débris basins will be expensive unless the hazard is eliminated at the source. The saving of evapotranspiration losses from trees removed from stream bottoms would pay the cost of removal. It would not be necessary to remove all the trees. A selection could be made and the trees with deep root systems and least water requirements, preserved for recreational purposes. Under the present policy of the Forest Service in closing most of these similar areas to recreational uses, in direct contradiction of the policy on which this Service was established, it would not be necessary to save even these few trees.

The maximum depth of débris deposition in the basin amounted to about 18 ft and was of such a mixture of mud, sand, gravel, and boulders that there would have been little drainable void space for underground storage, especially near the outlet. If the débris deposited behind a barrier is to be utilized for underground water storage it would be advisable to provide some means of passing mud during the flood.

In Article 5, the author gives two examples of the major débris-carrying streams in Southern California. A study of these data and those of other local streams would indicate that the grade of the débris cone just below the mouth of the canyon is an indication of the flow, the steeper cones being below the mouths of the streams of lesser flow. These examples could be used as a substantiation of the argument of Professor Huntington relating to profiles of débris cones in relation to stream flow.[47]

Mr. Sonderegger is clairvoyant in describing the results of dam failures on a débris cone (see paragraphs in Article 5 preceding "Rate of Débris Production"). These paragraphs, written as they were several months before the La Crescenta-Montrose flood, are prophetic. Pickens Canyon, having the major water-shed (1.6 sq miles) of the streams causing the damage, traverses a region of faulted and easily eroded rock. It has built a large débris cone below its mouth from previously eroded material.

The main branch of Pickens Canyon flows in a narrow trough, ranging from 50 to 100 ft deep. Before the flood it was crossed with a multitude of wire and rock check dams averaging 6 ft high. Most of these dams were founded on bed-rock bottom, or very near to it. The sides of the canyon, for a distance of at least ½ mile above its mouth, are very precipitous, and the soil is held in place by the large masses of roots of the oaks and other vegetation. The trees in the canyon bottom were not burned by the fire preceding the flood. About ½ mile above the mouth of the canyon there are typical slides which accompany faulting along such streams in Southern California. The bank material is loose rock with large masses of clayey earth. The rainfall of more than 12 in. for the storm, saturated this material, causing it to slump into the stream, carrying trees, rocks, and earth into the canyon bottom, where the trees lodged and formed temporary dams, behind which the heavier débris lodged. There would have been a débris flow from Pickens Canyon even if the water-shed cover had not been burned.

The check dams were washed out as soon as the confining wires were broken by the erosive action of the material in the stream. The débris accumulated behind these check dams was added to the slumped material and carried down the canyon. There was very little scouring of the channel below the original canyon bottom because of the shallow fill above bed-rock. At the mouth of the canyon, the stream debouches on a steep débris cone built up by previous floods. The stream channel across this cone above the highway was also crossed by numerous check dams. These dams accumulated large masses of débris before their failure, but when they failed, the channel was scoured

[47] "The Climatic Theory," by Ellsworth Huntington, *Publication No. 192*, Carnegie Institution, p. 33.

to depths ranging from 5 to 10 ft below the original bed. The hugh boulders deposited in the residential area below the State highway were carried down from this débris cone and their presence on the cone is evidence of similar but heavier floods in the past, before there was any over-grazing or forest reserve.

The check dams that withstood the flood in Haines Canyon, offer an example of what must have happened to the check dams on the various débris cones above La Crescenta and Montrose. Unlike Pickens Canyon, Haines Canyon has a broad flat débris-filled bed. The flood passing over the check dams in Haines Canyon scoured the channel below the toe of the dams and deposited the eroded material behind the next dam down stream.

The check dams on the Pickens Canyon débris cone remained intact long enough to raise the stream bed and cause overflow of the banks. Vast amounts of débris were then diverted out of the original channel. The culvert at the State highway was so well constructed that it did not wash out. Its capacity, however, was so reduced by a 6-ft boulder lodging at its entrance, that most of the débris coming down the channel passed over the highway, which acted as a spreading dam and spread the débris over a broad area.

The latest spreading works on the Upper Santa Ana River are designed to by-pass the silt back into the active channel, where the stream will have a flow diminished by the amount of absorption above. This action will tend to increase the silt load beyond normal conditions. When the stream reaches the absorptive area in the lower river any further absorption will raise the stream bed by the deposition of this silt. It is necessary, if spreading works are to be maintained at their maximum efficiency in such stream channels, that large flows be permitted to pass through them at intervals, in order that the silt may be scoured out, even if large flows may escape into the ocean. Professional conservationists, in their desire to impress the taxpayer with the importance of their endeavors, frequently cite this "large waste of water into the ocean" as the argument for more extensive works to preserve this water, even if the cost of the works outweighs the value of the water thus escaping. They fail to take account of Nature's laws and unwittingly accelerate the overthrowing of the physiographical balance.

J. C. Stevens,[48] M. Am. Soc. C. E. (by letter)[48a].—The author postulates his thesis on the theory that the physiographical factors combine to produce a balance. Such a general statement may well be challenged.

Nature herself seldom remains quiescent long enough to permit the establishment of anything like a permanent or even a gradually changing balance of these factors. She builds consistently for a few years and then with wild abandon utterly wrecks her handiwork. Man's efforts are puny in comparison, yet so conceited is he that he likes to believe that his is the mighty hand.

One factor at man's command is the construction of dams. Another is the straightening of stream channels. He may remove timber, cultivate lands, divert water for power and irrigation, and graze stock. Beyond these, his

[48] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[48a] Received by the Secretary April 2, 1934.

efforts are futile. Even his control is haphazard and uncertain, because his highest dams fail and his diversions get out of control, as witness the St. Francis Dam failure and the flooding of the Imperial Valley by the Colorado River. He cannot control, in the slightest degree, rains, frost, winds, tides, landslides, earthquakes, fire, and vulcanism. Nature buffets him mercilessly with these forces.

Man's works are temporary. Left without persistent and painstaking maintenance and repair, they soon become mere ruins, failing utterly in their intended purpose. Nature's forces never rest. These are such platitudes, it is almost a waste of space to recount them; yet, in the face of these facts, how can any one conceive of a physiographical balance "that undergoes only gradual changes"?

Was the Gros Ventre landslide (Wyoming), that put a dam in Gros Ventre River 1 000 ft high and, later, wrecked the entire landscape below it, a gradual change? Are the rather frequent Southern California floods caused by a foot or more of rainfall in a day factors for gradual changes? Is it fair or consistent, or rational, to assign the burning of a scattered growth of chaparral as the cause? If so, what about the recent unprecedented floods in the Pacific Northwest from the most heavily forested areas in this country? The changes effected were not gradual, but sudden and mighty. What about the Yellow River of China that, during the flood of 1851, suddenly left its channel, inundated an empire in spite of elaborate works of man, snuffed out a million lives, and found a new outlet to the sea 500 miles to the north? The entire physiographical complexion of China is thus suddenly changed every few generations. There is no balance.

A protracted wind storm a few years ago laid low thousands of acres of standing timber in Oregon. A more ruthless destruction, man, with all his machinery and ingenuity, has not equalled. In the summer of 1933 a forest fire on the west slope of the Coast Range in Oregon destroyed an enormous area of heavy virgin timber. That fire was carelessly set by man; yet, throughout the Northwest are untold acres similarly destroyed by fire from lightning before man's occupancy.

These few examples illustrate the impotency of man's control and raise a serious doubt as to whether any permanent relief can be predicated on a theory that there is a physiographical balance of Nature's factors. It seems that the author's physiographical balance is only Nature's quiescence between upheavals.

Erosion has come to claim the attention of scientists, pseudo-scientists, "yes men," and politicians. A struggle for the control of the grazing areas of the public domain has invaded Congress. Who gets that control has funds, personnel, jobs, influence. Propaganda in waves, broadcast by speeches, radio, articles in scientific and lay journals, and by newspapers, points to the same thing, "unprecedented and accelerated erosion due to over-grazing."

The writer believes in a National control of grazing and forested areas, but not for the reasons often assigned. Control of forest areas to perpetuate a timber supply and for recreational purposes is reason enough. The water-conserving properties are insignificant, unimportant, and enormously exag-

gerated. Control of grazing areas for maintenance of stock feed affords abundant reasons why it should be exercised. If grazing areas are so controlled for that purpose the erosion feature will be cared for automatically.

Erosion does not result from the mere removal of timber or the mere grazing of animals. It results from the cultural works of man. Roads, trails, and logways, are incipient causes of erosion. Field cultivation is another. On grazing areas erosion occurs at places of animal concentrations, such as waterholes, trails, and feeding and bedding grounds. On both forested and grazing areas such incipient erosion can be eliminated only in so far as these cultural activities can be eliminated or effectively controlled.

The writer agrees heartily with the author that temporary débris barriers, silt dams, flood channels, and works for erosion control of non-permanent materials are almost worse than none. When they fail, as they surely will, without well planned and well financed maintenance expenditures, the results are more disastrous than if they had never been constructed.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

MODEL OF CALDERWOOD ARCH DAM

Discussion

BY A. C. JANNI, M. AM. SOC. C. E.

A. C. JANNI,[21] M. AM. SOC. C. E. (by letter)[21a].—The care with which these tests were conducted, has produced results that deserve the attention of the profession. The writer has called attention,[22] in a general way, to the principal uncertainties attending the analysis of an arch dam, and these uncertainties seem to be confirmed, once more, by the tests and conclusions described in this paper.

The trouble is neither with the designer, nor his method of analysis; but with the assumptions he is compelled to make, and which he knows are not correct. Indeed, some of the assumptions are entirely contrary to the laws of equilibrium. Consequently, the results obtained by analysis, including the uncertainties that creep in with questionable assumptions, render them illusory, to a great extent.

The paper is concerned with measurements of the deformation of a rubber litharge model of Calderwood Dam, caused by hydrostatic pressure. From these measurements the authors reach the conclusion that the stresses that actually occur in arch dams are not the same as those determined by theory. The writer is in agreement with this conclusion. One of the reasons for this difference is that theory takes temperature effects into consideration, which cannot be measured by tests, because they do not deform the structure.

Every engineer knows that the stresses caused by temperature change in an arch dam are often more important than those due to the hydrostatic pressure. Dams are exposed to the most sudden and violent changes of temperature, and this fact is due to their location. For example, in a mountainous region, where dams are usually built, a sudden and important temperature change can occur almost over-night. Then the dam will have its down-stream face exposed to the surrounding temperature of the air, whereas its up-stream face is in contact with a body of water the temperature of which is quite different

NOTE.—The paper by A. V. Karpov and R. L. Templin, Members, Am. Soc. C. E., was published in December, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1934, by Messrs. A. W. Simonds and Lars R. Jorgensen.

[21] Cons. Engr., New York, N. Y.

[21a] Received by the Secretary March 8, 1934.

[22] "A New Type of Dam,'" by A. C. Janni, M. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., December, 1933.

from that of the air. In other words, while the up-stream face of the dam has a temperature of about 32° F, the down-stream face may have a temperature of, say, − 20° F. Every engineer can realize the unfavorable condition to which a concrete structure is subjected, when such temperature drops occur (as they often do).

Furthermore, the law of temperature transmission, in large masses of concrete, is not known, and this adds still more uncertainty to the analysis of temperature stresses. Assume, for example, that the temperature is the same for the entire structure, and that it is known. Although this assumption is almost certain never to occur, it will be made here, to simplify the argument.

Let t_e be the change of temperature at the extrados and t_i that at the intrados of the elemental arch of the dam. As a further simplification, assume that the law of transmission of temperature from the intrados to the extrados, or *vice versa,* is a linear one. (The latter assumption is known to be incorrect as applied to large masses of concrete.)

The static effect caused by a change in temperature may be considered as the algebraic summation of that caused by a uniform change, $\frac{1}{2}(t_e + t_2) = t_0$, plus that caused by a change, $\frac{1}{2}(t_e - t_i)$, at the extrados, and $-\frac{1}{2}(t_e - t_i)$, at the intrados. Assume that both t_e and t_i represent a temperature increase and that $t_e > t_i$. Let *AB*, in Fig. 14, be the elemental arch of the dam, and *CDEF* (Fig. 15), a voussoir of it. If End *A* were free, Section *CE*

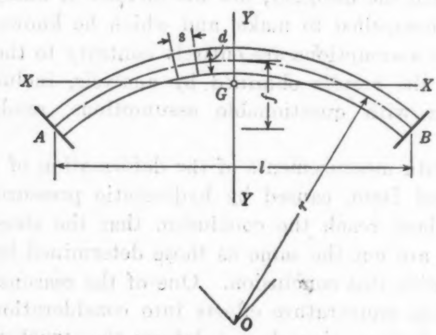

FIG. 14.

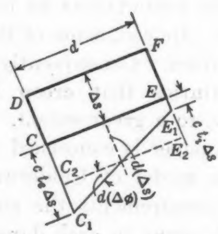

FIG. 15.

would take the position, $C_1 E_1$, after the arch expands, because $CC_1 = c\ t_e\ \Delta s$ and $EE_1 = c\ t_i\ \Delta s$. This total deformation consists of a displacement of the section, CE, to $C_2 E_2$, CE and $C_2 E_2$ being parallel (caused by the uniform change of temperature, $\frac{1}{2}(t_e + t_i) = t_0$, and by a rotation, $d\,(\Delta\phi)$), bringing the section, $C_2 E_2$, into the position, $C_1 E_1$, which is given by:[23]

$$d\,(\Delta\phi)\ \frac{c\,(t_e - t_i)\,\Delta s}{d} = \frac{c\,\Delta t\,\Delta s}{d} \ \ldots\ldots\ldots\ldots\ldots \quad (4)$$

The first deformation (to $C_2 E_2$) would cause a displacement of A (Fig. 14) to the left, an amount equal to; $c\ \dfrac{t_e + t_i}{2}\ l = c\ t_0\ l.$

[23] "Statica delle dighe per Laghi Artificiali," by C. Guidi.

However, End A of the arch is fixed and, therefore, there will be a reaction at that abutment of the same nature as that caused by the hydrostatic pressure. In other words, since Point A will not rotate, the reaction at that abutment will act along the X-axis and to the right (Point G being at the elastic center of the arch). The value of that reaction will be, $H_t = \frac{c\ t_0\ l}{I_x}$, which, after some substitutions and transformations, becomes: $H_t = E\ c\ t_0 \frac{d^3}{6\ \delta\ r^2}$, in which, all the letters have the usual meanings, and,

$$\delta = \frac{s}{l} + \frac{r - f}{r} - \frac{2\ l}{s} + \frac{d^2}{6\ r^2}\left(2\ \frac{s}{l} - \frac{r - f}{r}\right)$$

Concerning the second part of the deformation, it is observed that if End A were free, then it, together with the elastic center, G (assumed to be rigidly connected with A), would rotate through an angle given by Equation (4), when that equation is extended to the entire arch, that is: $\Delta\phi = c\ \Delta t\ \Sigma\ \frac{\Delta s}{d}$.

At the same time, Center G would be displaced in a direction, the components of which, along the X and Y-axes, respectively, are:

$$\Delta x = c\ \Delta t\ \Sigma\ \frac{\Delta s}{d}\ y \qquad (5)$$

and,

$$\Delta y = c\ \Delta t\ \Sigma\ \frac{\Delta s}{d}\ x \qquad (6)$$

Since d is constant for the entire arch, the center of gravity of the arch will coincide with the elastic center, G (center of gravity of the elastic weights of the entire arch). In fact, the elastic weight of an element of the arch is given by $dG = \frac{ds}{E\ I}$, in which, $I = \frac{1}{12} \times 1 \times d^3 =$ a constant; therefore, dG is proportional to ds; and the two centers of gravity of the arch will coincide. Because of this fact, Equations (5) and (6) will give:

$$\Sigma y\ \Delta s = \Sigma x\ (\Delta s) = 0 \qquad (7)$$

and,

$$\Delta y = \Delta s = 0 \qquad (8)$$

Equation (8) reveals that, during the second part of the deformation of the arch to $C_1\ E_1$ (Fig. 15), if End A were not fixed, it would rotate about the elastic center, G. However, End A is fixed and, therefore, it will develop a reaction which, according to the theory of the ellipse of elasticity, will be a force infinitely small and infinitely far from Abutment A; that is, a couple, M. Since, according to the theory of the ellipse of elasticity, $\Delta\phi$ is always equal to $G\ M$, the rotation will be given by:

$$M = E\ \frac{\Sigma d\ (\Delta\phi)}{\sum \frac{\Delta s}{I}} \qquad (9)$$

In the presence of Equation (4), with d constant:

$$M = \frac{E\,I\,c\,\Delta t\,\Sigma\,(\Delta s)}{d\,\Sigma\,(\Delta s)} = \frac{E\,I\,c\,\Delta t}{d}$$

but, since $I = \dfrac{d^3}{12}$:

$$M = \frac{E\,c\,\Delta t}{12}\,d^2 \quad \text{.........................(10)}$$

Thus far, this discussion has been concerned with an analysis of stresses due to a temperature change as it applies to the conclusions offered by Messrs. Karpov and Templin. Next, consider the nature of the deformation caused by the foregoing temperature stresses.

As stated previously, the effect upon the dam of that part of change in temperature, which has been considered as uniform (to $C_2\,E_2$, in Fig. 15), is perfectly similar to the effect upon the dam caused by hydrostatic pressure. As in the latter case, the unit shortening of the geometrical axis of the arch is given by $\dfrac{p\,r}{E\,d}$, in which, p = hydrostatic pressure, and r = radius, so, in the case of a uniform change in temperature, t_o, the same axis will be translated a unit distance, caused by $\pm\, c\, t_o$.

Therefore, if the equation for the deflection of the crown of the elemental arch of the dam, caused by hydrostatic pressure, is:[24]

$$\Delta f_p = \frac{p}{E}\,\frac{r}{d}\,f\left\{1 + \frac{1}{\delta}\left[2\,\frac{l}{s} - \frac{l^2}{4\,f\,r}\left(1 - \frac{d^2}{6\,r^2}\right)\right]\right\} \quad \text{.....(11)}$$

and if $c\,t_o$ is substituted for $\dfrac{p\,r}{E\,d}$, the final result will be:

$$\Delta f_t = \Delta f_p\,\frac{E\,c\,t_o\,d}{p\,r} \quad \text{..................... (12)}$$

which is the formula for the vertical deflection of the crown of the arch, caused by the uniform change in temperature.

For the second part of the deformation (to $C_1\,E_1$, in Fig. 15), the deflection of the crown of the arch is zero. In fact, imagining, again, that End A of the arch is free, the two sections, CE and DF (Fig. 15), would rotate, with respect to each other, at an angle, $d\,(\Delta\phi)$, as given in Equation (4); but End A is fixed, which creates a couple that acts upon every element of the arch; and the value of its moment is given by Equation (10). Furthermore, the rotation of the two sections, CE and DF, is given by:

$$d\,(\Delta\phi) = \frac{M}{E\,I}\,\Delta s = \frac{\dfrac{E\,c\,\Delta t\,d^2}{12}}{E\,\dfrac{d^3}{12}}\,\Delta s = \frac{c\,\Delta t\,\Delta s}{d}$$

which expression coincides with Equation (4), and demonstrates that the elastic deformation caused by the reaction at End A is equal and opposite to

the thermal deformation. This fact is very important because it demonstrates that, although stresses are developed, the geometrical axis of the arch does not deform.

Obviously, models are useless for the determination of these last stresses; on the other hand, while theory shows that those stresses may be very important, the assumptions upon which the theory is based are known to be uncertain. At first, this phenomenon may appear inconsistent; but if a cube of concrete is imagined as held firmly between the plates of a compression machine, and if by some means the temperature of the cube is increased, the concrete is then known to be subjected to considerable compression, whereas its height is not changed.

By their conclusions, the authors have given an additional warning to the profession; and they have made a worthy contribution to the literature on dams.

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DISCUSSIONS

LOSS OF HEAD IN ACTIVATED SLUDGE AERATION CHANNELS

Discussion

BY MESSRS. HENRY R. KING, AND M. H. KLEGERMAN

HENRY R. KING,[5] ESQ. (by letter)[5a].—This is the first publication of tests on the loss of head in aerated channels carrying activated sludge, that has come to the writer's attention. Such tests aid in the selection of economical channel sizes and are particularly helpful in the design of an aerated influent channel for a large battery of tanks, so that an equal division of the flow may be assured among the tank units with a minimum amount of regulation of the inlet gates.

The scarcity of data on the subject can be attributed to several reasons. The activated sludge process is relatively new; large plants are few; and, consequently, there is a lack of data on many special problems of design. The head losses occasioned by aeration constitute only a small part of those occurring in a plant as a whole. Of major importance are the losses at tank inlets and exits, through meters and bends, in passing over weirs, and in the conduits between the several stages involved in the complete treatment of the sewage.

The designer may be 100% in error in estimating the additional friction of flow due to aeration, but the hydraulic capacity of the plant will be approximately as originally intended. Consequently, the need of experiment may not have been fully appreciated. If, however, it was, the fact remains that the aeration channels in an activated sludge plant are not as a rule very suitable for experiments. The velocities are low, the distances are short through which uniform flow obtains, and, consequently, the head losses are so slight as to make precise measurement difficult, inasmuch as measurements are required where the flow is disturbed by numerous rising air bubbles and where the variation in the porosity of the diffuser plates may result in a greater disturbance near one gauge than at another. However, the writer believes that tests can be made to secure worth while results.

NOTE.—The paper by Darwin Wadsworth Townsend, M. Am. Soc. C. E., was published in January, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1934, by H. L. Thackwell, M. Am. Soc. C. E.

[5] Asst. Civ. Engr., The Sanitary Dist. of Chicago, Chicago, Ill.

[5a] Received by the Secretary April 9, 1934.

It would have been of interest if Mr. Townsend had given a description of the procedure followed in making the tests at Milwaukee. Perhaps the floats would have been less disturbed by pulsations if the upturned entrances of the pipe connections to the float-tubes had been lower down in the channel than shown in Fig. 1. The weight of the liquid on the bottom of the channel remains constant regardless of the volume of entrained air. At an elevation some distance above the bottom, as the volume of air in the mixture increases, a greater proportion of the weight of the liquid is lifted above this elevation and, as a consequence, the pressure here increases. Therefore, one would presume that the nearer the entrances of the piezometer piping to the bottom of the channel the less the magnitude of the pulsations in the float-tubes, due to variations in the volume of air in the mixture.

The following procedure is suggested for making tests that involve relatively small losses, such as Mr. Townsend had at Milwaukee (0.02 to 0.03 ft). Care should first be taken to determine the inherent errors in the gauges and, before each test, they should be checked with the channel standing quiescent at the normal depth of flow. Then, the air should be turned on at the rate to be used during the test and the effect of aeration at zero velocity noted at the gauges. The flow should then be started and maintained at a constant rate for some time while the losses are recorded first with the air on and then with the air off. The difference in head loss for these two conditions of flow would, of course, be that due to aeration.

In 1931, at the North Side Sewage Treatment Works, of The Sanitary District of Chicago, head-loss tests were made on a return sludge aeration channel, 4 ft wide, with vertical walls. On the bottom, on the center line, was one longitudinal row of diffuser plates (12 by 12 in.), in boxes, 18 in. wide, from which the bottom sloped upward at 45° to meet the channel walls. The return sludge flowed at depths between 4 and 5 ft above the plates. The hydraulic radius was approximately 1.4 ft and the rate of aeration, 1.4 cu ft of free air per min per lin ft of channel. The test was made over a length of 410 ft, in which the flow increased by five equal increments, each introduced from two pipes at right angles to the longitudinal axis of the channel about 6 in. above the surface and at intervals of about 70 ft. For 20 000 000 gal per day the velocity varied from 0 to 2.25 ft per sec, and the surface drop for the entire distance was 0.42 ft. Making allowances for head losses due to the falls of incoming sludge, in accordance with the method devised by Julian Hinds, M. Am. Soc. C. E., for side channel spillways,[8] the value of n in the Kutter formula was computed at 0.034, or more than twice the value for the same sludge in a closed conduit. This friction coefficient seemed high. To determine whether this was due to the use of air, another test was made for a total quantity of 20 000 000 gal per day. Stable flow was maintained while measurements were repeated several times, with the air alternately on and off.

The results showed an average surface drop of 0.42 ft with aeration and 0.41 ft without it. This test indicated that aeration had but slight effect on head loss in this particular case. Hence, the high loss must have been caused

[8] "Side Channel Spillways," by Julian Hinds, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 881.

by the obstructions presented within the channel itself through the presence of the air pipes and the disturbances caused by the downward discharge of the pairs of incoming sludge pipes. Although the channel was far from ideal for the purpose of determining accurately the effect of aeration on head loss, the writer is inclined to believe, from the results obtained, that the effect may be relatively small when compared to the losses that occur without the presence of aeration.

M. H. KLEGERMAN,[7] JUN. AM. SOC. C. E. (by letter)[7a].—Further evidence that engineers engaged in the design and operation of hydraulic works are seeking and finding information concerning hydraulic coefficients, either to re-assure themselves of the work of the early investigators, or to find applications for new conditions, is presented by the tests performed at Milwaukee, Wis., and described in this paper.

Many of the empirical data upon which design to-day is based, had their beginning, as is generally known, in the middle of the Eighteenth Century (Brahms, 1753; Brahms and Chezy, author of the well-known formula, $V = c\sqrt{RS}$, in 1775), and many of the coefficients often accepted by engineers at the present time, without much concern as to their limitations or applicability to modern materials, were developed by Ganguillet and Kutter and others, in the latter part of the Nineteenth Century. In the course of the translations of the work of Ganguillet and Kutter, additional important contributions were made to the hydraulic literature on the subject of frictional coefficients by the various translators. Probably an outstanding contribution since the beginning of the Twentieth Century, can be found in the work of Hazen and Williams, in the development of their formula, now so well known, for the determination of flow in pipes, together with the publication of their hydraulic tables presenting frictional coefficients and head losses.

In the water-works field particularly, the comparatively recent introduction of new material (bituminous and cement coatings for steel and cast-iron pipes, used both for protection against corrosion and for increasing the carrying capacity by providing a smoother interior surface; the greater use of concrete pipe; and the recent introduction in the United States of such new pipe-manufacturing methods and materials as centrifugally spun cast-iron pipes, modern steel and concrete pipe, and pipe of a cement and asbestos compound) has created keen interest among engineers as to the question of coefficients of flow and their interpretation as to capacity.

The technical literature of other countries, particularly Great Britain, also discloses that the relatively recent departure in pipe lining from the original practice of merely dipping cast-iron or steel pipes in a coal-tar and linseed-oil preparation, is directing considerable interest among European engineers to the necessity of obtaining a formula for the discharge of these new types of pipe. It is the opinion that without such a formula it is not feasible with certainty to watch the rate at which discharge is diminishing, since no common datum is available on which to base later results.

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[7] Asst. Engr., Alexander Potter, Cons. Engr., New York, N. Y.

[7a] Received by the Secretary April 21, 1934.

More information, too, is now sought not only on carrying capacities of conduits of these materials when new, but also on the relative ability to maintain initial carrying capacities and their rates of reduction, under various conditions of usage.

While engineers who have the facilities and opportunities at hand for making these tests are engaged upon such work, it would seem highly desirable to extend it into an examination of conditions of flow in which not only water is concerned, but in which water is in an intimate physical combination with other substances, such as air, in the case of aeration channels, or sludge, in the case of sludge force mains. Furthermore, conditions about sewage treatment plants present many cases of varied or unsteady flow, even where sewage alone is concerned. Instances of test data for head losses under such conditions, are not many, and an organized attempt to obtain and correlate mass data of this kind would prove of value to designing engineers.

The tests described by Mr. Townsend indicate that head losses occur to an extent probably not before fully realized, and must be considered in the design of channels of the type described. It is unfortunate that the tests had to be made on a comparatively short length of channel (the length being limited only to 123.34 ft), because, as stated in the paper, the range in pulsation of the recorders was from three to four and one-half times the loss in head over the section. In view of this condition, it is difficult, immediately, to accept the conclusion that the "final results stated in the paper are sufficiently on the side of safety to warrant confidence in their subsequent use." The likelihood of finding aeration channels of considerable length for testing purposes, is probably very limited, however.

It is noted that the value of the slope used for computations in Kutter's formula, was that of the water surface as determined from the depth recorders established at each station. In an open channel of the type described, it is likely that disturbances introduced by the injection of compressed air, atmospheric disturbances, irregularities of the channel and of the diffuser plates, etc., may have resulted in different depths of flow and, consequently, in different velocities at each of the observation stations. In that event the slope of the energy gradient, instead of the water surface, would result, of course, in more accurate determinations. The actual effect upon the value of n as so determined, however, can not be known without the information required to compute the energy gradients at each station.

If other tests of this nature should be undertaken, it would be of value to know what the temperature conditions and atmospheric pressures are during the time of test, as these conditions may influence air-bubble coalescence which, as the author described, has an important bearing on the resistance to flow. In open channels of the type used in the Milwaukee test, some resistance to flow must certainly be created by wind resistance at the water surface exposed to the atmosphere. In tests performed upon large sewers flowing partly full, the writer has found that values of n are somewhat higher (due, it is believed, to "air resistance" at the surface) than when flowing full or

nearly full. Observations on the direction and intensity of wind during tests, therefore, might be of some value in comparisons of test data.

"Before-and after" tests—that is, determinations of head loss with and without aeration—are important in order that the effect of the latter may be fully evaluated. The author has apparently obtained such information, as he gives the value of $n = 0.013$ for the channel with non-aerated sewage, although it is not clear whether this value has been determined from tests with diffuser plates in position, or whether it is an assumed condition.

Information as to surface and bottom velocities, as well as on mean velocities and their inter-relation, would perhaps help determine to what extent the flow varied from "free-water" flow, and, also, in what sections of depth the greatest disturbances occur.

The varying value of n with varying velocities, as reported by the author (that is, $n = 0.03$ at a velocity of 1.3 ft per sec, with $n = 0.23$ as velocity approaches zero), indicates, of course, that the condition of resistance to flow as denoted by n, is "internal" in Nature; that is, it more nearly represents a state of the contained liquid under varying conditions, than a condition of the channel itself, with which Kutter's n is usually associated. While the author brings out the fact that for the purpose of his paper, the subject is treated from the viewpoint of an increased roughness coefficient, assuming the flowing medium to possess the hydraulic properties of water alone, it would appear of interest and along the methods often adopted in estimating transition losses, etc., to learn whether or not head losses for a flow of an air and sewage mixture could not be more practically expressed in terms of "percentage of loss of head for clear water." Expressed in these terms, there may be the further advantage of eliminating the use of a varying coefficient.

There is another explanation, however, that might be made in connection with the author's conclusion that the coefficient of roughness, n, increases as the velocity decreases.

By reference to Fig. 1, it will be observed that the channel in which the tests were performed is composed of two different materials, namely, the sides of concrete and the bottom of a combination of concrete and diffuser plates. In other words, the bottom and sides of the channel are of different degrees of roughness; the roughness coefficients for the channel bottom and sides have different values.

Robert E. Horton, M. Am. Soc. C. E., refers[8] to this condition as "composite" roughness. According to Mr. Horton,

> "The value of Kutter's or Manning's n applicable to the sides of such a channel may be designated as n_s and that applicable to the bottom as n_b. The value of n derived from experiments on such a channel may be designated the 'equivalent' roughness. If a channel had a degree of roughness corresponding to n throughout its entire cross-section, it would give the same discharge and velocity as the actual or experimental channel at the same depth and slope."

[8] *Engineering News-Record*, November 30, 1933.

Mr. Horton further established the fact that a value of equivalent roughness (assuming, of course, constant "internal" characteristics of the flowing liquid) derived from experiments on a given channel is really applicable to a single depth. In other words, if the roughness of the bottom is not the same as that of the channel sides, the equivalent value of n varies with the depth.

A formula suggested by Mr. Horton for determining the value of n applicable to channels with two degrees of roughness, is as follows:

$$n = \left(\frac{n_b^{\frac{3}{2}} + 2\,z\,n_s^{\frac{3}{2}}}{1 + 2\,z}\right)^{\frac{2}{3}} \quad \text{.........................(3)}$$

in which, z = ratio of depth of flow in channel to bottom width.

As is evident from the foregoing, the composite n will vary with z, so that if $z = 0$, $n = n_b$, and n approaches n_s as z approaches infinity.

In the absence of more specific observational test data, it is not possible to state whether the results obtained by Mr. Townsend would agree fully with Equation (3). The original observations may contain sufficient data so as to make possible the computation of n_b from this equation (assuming n_s to be equal to 0.013.) Having computed n_b for a given value of z and n, it would then be a simple matter to substitute in Equation (3) the value of n_b thus obtained, and with a different observed value of z, compute the value of the composite n. Comparison of the value of n thus computed with that reported for a specific velocity, might establish whether or not the relation implied by the Horton formula holds in the case of aeration channels.

The author is to be commended for his work and method of presenting results. Of particular interest is the presentation of the "cause and effect" theories. His efforts will undoubtedly result in directing attention to this phase of sewage flow and should encourage others to investigate and publish their results for the benefit of all engaged in the design of sewage works.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

RAINFALL STUDIES FOR NEW YORK, N. Y.

Discussion

BY MESSRS. H. ALDEN FOSTER, J. J. SLADE, JR., CHARLES W. SHERMAN, MERRILL M. BERNARD, CLIFFORD SEAVER, AND JOSE GARCIA MONTES, JR.

H. ALDEN FOSTER,[3] M. AM. SOC. C. E. (by letter)[3a].—The author has analyzed the rainfall data by probability methods, based on the frequency curves of Karl Pearson. As no details of this analysis are presented, it may be helpful to engineers who are unfamiliar with these methods to state that the subject was treated, in 1924, by the writer[4] and, in 1927, by R. D. Goodrich, M. Am. Soc. C. E.[5]

In applying the frequency curve method, the author states that "the coefficient of skew appears to vary from 1.56 to 3.02, with an average value of 2.22." Fig. 5 was plotted, using the average coefficient of skew, 2.22, for all the curves. It is apparent from an inspection of Fig. 5 that this average value is not suitable for all the curves. It is not surprising, therefore, that the probability method used by Mr. Bleich gives results that differ from the actual data more than those obtained by means of the exponential method (Fig. 3, or Equation (5)).

The author states that "it is desirable to harmonize all the data given in Table 3 showing a relation between R, t, and n," and submits Equation (8) as a summary of these data. In his conclusions he states that "Equation (8) does not appear to be sufficiently close to the observations to warrant its use for all kinds of storms." A generalized empirical formula is a dangerous tool to use in any case, unless its limitations are well understood. It would seem unwise, therefore, to publish a new rainfall formula that is admittedly unreliable even when applied to its basic data.

NOTE.—The paper by S. D. Bleich, Esq., was published in February, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[3] Washington, D. C.

[3a] Received by the Secretary, February 23, 1934.

[4] "Theoretical Frequency Curves and Their Application to Engineering Problems," by H. Alden Foster, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142.

[5] A graphical method for analyzing similar data is given in "Straight Line Plotting of Skew Frequency Data," by R. D. Goodrich, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 1.

The rainfall records given in Tables 2 and 3 are well prepared, and should be of value to the profession. The formulas in Table 7 should also be useful as a guide for engineers studying problems of storm run-off in this part of the United States.

J. J. SLADE,[6] JR., ESQ. (by letter)[6a].—An interesting and important correlation between the duration and intensities of New York storms is developed in this paper. The author has spared no labor in comparing the results obtained by the use of various methods of analysis, and he finds that his formula gives a better representation of the observed data than others that he considers.

One question is left open, however, which merits consideration: What are the values that any formula is expected to give? The observed data represent a set of events that have occurred in the past and that almost certainly will never occur again in the form once observed. The fact that an empirical formula gives, closely, the values of an observed set of events does not mean that it will fit closely a future set. It is not surprising that the author finds that the values obtained from his modified exponential formula agree more closely with observed values than do those obtained by the probability method, since he has determined the constants in his formula by the method of least squares to give just such an agreement.

Mr. Bleich regards the probability method as an alternate in the solution of this problem. The method is not alternative, however, but rather complementary to any method used. The correlation required in this particular problem is between members of a small set culled from a large group of events. Such a local set cannot be treated independently; it belongs to the whole group, and it is only as a part of the whole group that one may make assertions about it.

The probability method gives information about the entire group of events. For instance, it involves all the available information regarding the 30-min duration intensities, and then it indicates how, in the long run, these intensities will be distributed. It does not fit the record locally; it fits it as a whole. The past 62-yr record deviates from the probability curve, and so will the future record; but the probability curve furnishes a norm about which any record of the phenomenon in question will fluctuate.

A glance at Fig. 6 shows that, locally, all the duration curves deviate from the probable norm in much the same manner. That the deviations should be so uniform is not surprising when one remembers that the grouping of the durations into the various classes is to a great extent artificial, and that these various classes really blend into each other. On the other hand, this uniform deviation may be taken to mean that the actual distribution of intensities is not the one given by the probable norm, but rather by some other curve which should fit the observed data more closely—only the observations of the next few hundred years will give an answer to this question. However, it is likely that this deviation is characteristic of the current meteorological period more than of the long-term distribution of intensities.

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[6] Asst. Prof. of Eng. Mechanics, Rutgers Univ., New Brunswick, N. J.

[6a] Received by the Secretary March 2, 1934.

If these contentions are correct, the constants of the modified exponential formula should be computed, not from the observed data, but from the modified values given by the probability method, because the determination of the constants directly from the data makes use of only a few isolated observations.

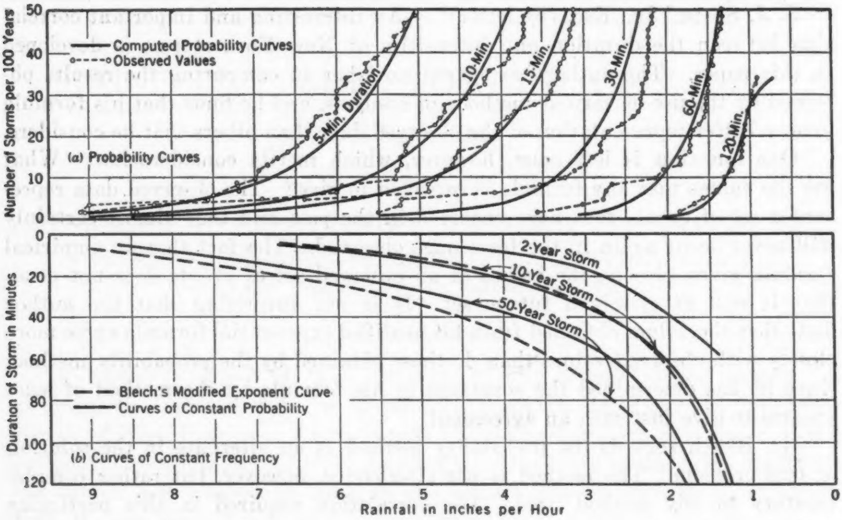

FIG. 6.—CURVES SHOWING THAT, LOCALLY, ALL DURATION CURVES DEVIATE FROM THE PROBABLE NORM IN MUCH THE SAME MANNER.

It does not in any way take into account the distribution of other intensities, whereas the values as adjusted by the probability method result from taking into account the entire observed set.

Fig. 6(*a*) gives the probability curves for the various durations carried out to a frequency of 50 storms in 100 yr. These curves are the same as those in Fig 5. The slight modifications result from the fact that the writer has used the true coefficient of skew for each curve instead of the author's constant value of 2.22. Fig 6(*b*) gives three curves of constant frequency as obtained from the curves of Fig 6(*a*), together with two of Mr. Bleich's modified exponential curves. It is seen that the 50-yr storm curve would be greatly modified by the suggestion made in this discussion. The curves have been plotted on ordinary co-ordinate paper so that they may be seen in their true relation to each other.

CHARLES W. SHERMAN,[7] M. AM. SOC. C. E. (by letter)[7a].—The writer has been greatly disappointed in Mr. Bleich's paper. He cannot but feel that the author has failed to analyze thoroughly the records of the automatic raingauge, which has been longest in use of any in the United States, and to derive results that will be of real value. It is especially discouraging to find that, apparently, Mr. Bleich has not even utilized all the data published[8] by

[7] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[7a] Received by the Secretary March 5, 1934.

[8] *Engineering News*, 1916, pp. 76, and 594.

O. Hufeland in 1916, and that, for years later than 1913, he has contented himself with the incomplete data published by the U. S. Weather Bureau, instead of compiling adequate data from the original chart records.

Under the circumstances, the writer will content himself with a brief statement of the principal points in which he feels that Mr. Bleich's paper is open to criticism:

1.—There is no description of the gauge and its exposure, and there is no reference to any other source from which such information can be obtained. Neither is there any comparison of records from this gauge and other gauges in the vicinity to indicate the accuracy of the record.

2.—No records of precipitation for periods between 30 and 60 min, or between 60 and 120 min, are given, nor are there any for durations exceeding 120 min.

3.—There is no indication that the author has given any consideration to what the writer has called the "Extended Duration Principle."[9] It is quite possible that a considerably larger number of records of storms of comparative long duration than the author has utilized, would have been obtained by the application of this principle.

4.—It does not appear logical that the constant, C, in Equation (5), should vary from 38.85 for storms of 1-yr frequency, to 4 201 for storms of 50-yr frequency; nor that the exponent, e, in the same formula should vary between 0.795 and 1.513.

In his own studies of rainfall intensity and frequency at Boston, Mass., the writer found[10] that C varied according to a mathematical law from 16.0 for 1-yr frequency to 45.9 for 50-yr frequency; and that d and e were constant. Similar results, although with other values of the constants, were found[11] for Detroit, Mich., by Milton F. Wagnitz, M. Am. Soc. C. E., and Lewis C. Wilcoxen, Assoc. M. Am. Soc. C. E.

Equation (8) is similar in form to that used by the writer and by Messrs. Wagnitz and Wilcoxen. Apparently, Mr. Bleich has not much confidence in it; but the writer suspects that if the analysis of the records had been more detailed, as suggested in the preceding Comments (2) and (3), considerable justification for Equation (8), or one similar to it, would have been found.

A comparison of this formlula with those devised for Detroit, and for Boston, is as follows:

Place	Formula	Derived by:
New York, N. Y.	$Rn^{0.3} = \dfrac{42.5}{(t + 12)^{0.85}}$	Bleich
Detroit, Mich.	$Rn^{0.263} = \dfrac{37.6}{(t + 8)^{0.855}}$	Wagnitz and Wilcoxen
Boston, Mass.	$Rn^{0.27} = \dfrac{16.0}{(t + 7)^{0.7}}$	Sherman

[9] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 953.
[10] *Loc. cit.*, p. 955.
[11] *Loc. cit.*, p. 965.

5.—Mr. Bleich has made no mention of previous papers on rainfall intensity as indicated by the Central Park gauge, except that of Mr. Hufeland.[8] The data and discussions in the reports of the Committee on Rainfall and Run-Off of the Municipal Engineers of the City of New York, published in 1913 and 1922, are worthy of consideration and comment, as are also comments[12] by the late Kenneth Allen, M. Am. Soc. C. E., in 1921, and again in 1922.[13] Mr. Allen's formulas are of the same general form as those of Mr. Bleich, as given in Table 7; but the constants given by Mr. Allen differ from those presented by Mr. Bleich much more than would be expected as the result of including ten years of additional records. It would seem that some statement regarding the earlier studies and explanation for the different results obtained would be in order.

Merrill M. Bernard,[14] M. Am. Soc. C. E. (by letter)[14a].—This paper is to be commended as a presentation of an excellent procedure in the treatment of rainfall intensity data. Mr. Bleich has accomplished the task of fitting curves, accurately, to plotted points and developing the mathematical expression for the curves; but can the formulas for the 25-year storm and the 50-year storm be accepted as the permanent relationship of rainfall rate and duration? Will the next 63 years of record reproduce the same curve group? When the station has reached 80 years of record, will not several of the present maximum values remain unexceeded? Furthermore, is it not probable that the next 40 years of the record will contribute a number of values which will fall within the unusual interval between the author's 10-year and 25-year storm, thereby affecting the position of the higher frequency curves? What is the climatological cycle, in years, necessary to be spanned by a single station record, in order to make that record a dependable basis for the upper frequency values? How accurately does the composite record fulfill the purpose of the single station record?

The writer believes that the formula types, reciprocal and exponential, serve two distinct purposes, the former more accurately representing the comparatively short periods of continuous downpour and the latter the average of intermittent rainfall for the longer durations. His opinion has been that, because of a paucity in single-station records of adequate length, covering duration periods of less than 1 hr or 2 hr, the Meyer curve group,[15] properly interpolated geographically, could be accepted as the best available expression of rainfall intensity for such duration periods. That New York City could have adopted them without material error is shown subsequently.

The rainfall intensity formula for New York City, taken from the writer's charts,[16] and designed for durations of from 120 min to 4 days, is,

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[12] *Engineering News-Record*, 1921, Vol. 86, p. 588.

[13] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 119.

[14] Cons. Civ. Engr. (Bernard Engrs., Inc.), Crowley, La.

[14a] Received by the Secretary March 9, 1934.

[15] "Elements of Hydrology," by Adolph F. Meyer, M. Am. Soc. C. E., Second Edition, pp. 197–198.

[16] *Transactions*, Am. Soc. C. E., Vol. 96, (1932), p. 618, Figs. 18, 19, and 20.

$$R_a = \frac{28\,F^{0.22}}{t^{0.75}} \qquad (9)$$

in which, F is the frequency, in years, and t, the duration of rainfall exceeding 1 hr, in minutes. To conform with Mr. Bleich's notation the writer's i is given as R_a, the average rainfall intensity, in inches per hour.

It should be noted that Equation (9) is developed through researches which have utilized the theory that a composite record of several stations within a limited area may be taken as a single-station record having a length equal to the sum of their station-years.

If the formula is modified by the inclusion of an additive factor in its denominator, it can be brought into a general form that expresses rainfall rate in terms of any duration, from 5 min to several days, and for any frequency. This factor seems to be a function of frequency, and expressing it thus, the formula for New York City becomes,

$$R_a = \frac{28\,F^{0.22}}{\left(t + \dfrac{9}{F^{0.20}}\right)^{0.75}} \qquad (10)$$

Fig. 7 is a comparison of the resulting curve-group and the plotted curves of Mr. Bleich's study.

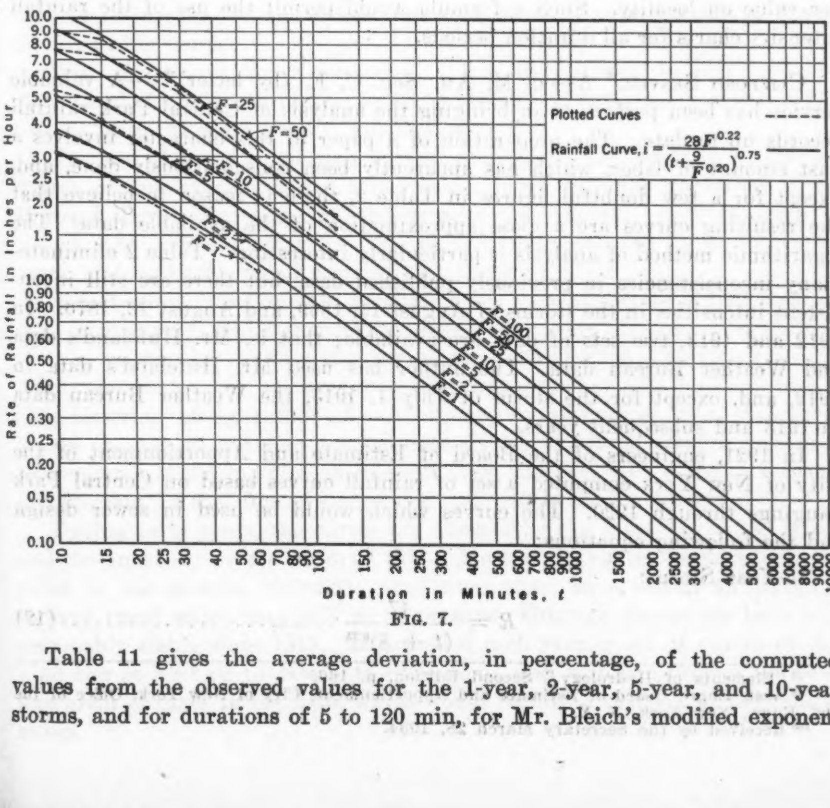

FIG. 7.

Table 11 gives the average deviation, in percentage, of the computed values from the observed values for the 1-year, 2-year, 5-year, and 10-year storms, and for durations of 5 to 120 min, for Mr. Bleich's modified exponen-

tial formulas, the writer's formula for New York City, and the Meyer formulas, in which the factors are given the average of the Group 2 and Group 3 values.[17]

TABLE 11.—COMPARISON OF AVERAGE DEVIATION FROM OBSERVED VALUES FOR THREE TYPES OF RAINFALL INTENSITY FORMULAS

Storm	Average Deviation from Observed Values, Percentage		
	Author's modified exponential formulas	Equation (10)	Meyer's reciprocal formulas
1-year	1.50	3.39	7.26
2-year	2.27	2.21	7.03
5-year	1.49	1.51	4.07
10-year	2.29	5.79	3.40

The writer has sacrificed little in curve and point agreement to maintain the general form of his equation. A more general form suggesting itself is,

$$R_a = \frac{K\,F^x}{\left(t + \dfrac{B}{F^x}\right)^n} \qquad (11)$$

in which, K and B are coefficients and x and n are exponents, all depending for value on locality. Such a formula would permit the use of the rainfall intensity charts for all duration periods.

CLIFFORD SEAVER,[18] ASSOC. M. AM. SOC. C. E. (by letter)[18a].—A valuable service has been performed in bringing the analysis of Central Park rainfall records up to date. The preparation of a paper of this character involves a vast amount of labor, which has apparently been conscientiously done, and, except for a few doubtful figures in Table 2, there is reason to believe that the resulting curves are a close approximation of the available data. The logarithmic method of analysis is particularly interesting. Table 2 eliminates many inconsistencies in previously published data, but there are still inconsistent intensities in the storms of August 15, 1869, and August 16, 1876. In 1912 and 1913, two sets of data are available; that is, Mr. Hufeland's data and Weather Bureau data. The author has used Mr. Hufeland's data in 1912, and, except for the storm of July 1, 1913, the Weather Bureau data in 1913 and subsequent years.

In 1921, engineers of the Board of Estimate and Apportionment of the City of New York computed a set of rainfall curves based on Central Park gaugings through 1920. The curves which would be used in sewer design had the following equations:

15-Year Storm:

$$R = \frac{47}{(t + 8)^{0.75}} \qquad (12)$$

[17] "Elements of Hydrology," Second Edition, p. 166.

[18] Asst. Engr., Board of Estimate and Apportionment, City of New York, Office of the Chf. Engr., New York, N. Y.

[18a] Received by the Secretary March 28, 1934.

10-Year Storm:

$$R = \frac{43}{(t + 8)^{0.75}} \quad \text{.........................(13)}$$

5-Year Storm:

$$R = \frac{25}{(t + 4)^{0.67}} \quad \text{.........................(14)}$$

It was the intention to use the equation for the 15-year storm for the most highly developed areas, and those for the 10-year and 5-year storms for partly developed territory. The writer had no part in preparing these curves and does not know how much mathematical refinement was used in fitting them to the data, but it is interesting to note that Equations (12), (13), and (14) are of the "modified exponential type" (Equation (5)) found by the author to fit his data best.

For a given frequency, the curves developed by the Board of Estimate and Apportionment are somewhat lighter than those proposed by the author, the Board's 10-year storm being approximately equal to the author's 5-year storm. This is due, primarily, to a different conception of frequency, rather than to any marked change in the data since 1920. The Board's engineers defined a 5-year storm as one which, over a 50-year period, would not be equalled during 40, or exceeded during 41, of those years. They placed no limits on the number or intensity of the cloudbursts that might be expected during the remaining nine years.

With this conception of frequency, it is a little difficult to define a 1-year storm, and, for this reason, the writer is inclined to favor the author's definition as being of more general application.

The use of a set of rainfall curves, each curve applicable to a different degree of development, never found much favor with the engineers of the Borough Sewer Bureaus. At the present time (1934), each Borough of the City of New York has its own rainfall curve, selected by its own engineers, and applicable to all parts of that Borough.

A fact sometimes forgotten by those using rainfall curves is that they are of a temporary nature and must be revised from time to time, if they are to remain reasonably accurate in view of all existing data. To illustrate this point, the writer has prepared two diagrams, which might be described as "life histories" of the author's 5 and 10-year rainfall curves. Fig. 8(*a*) shows yearly changes in the data for the author's 5-year storm, assuming them to be computed at the end of each calendar year, and Fig. 8(*b*) is the corresponding diagram for the 10-year storm; the curve labels are the durations.

During early years, the curves are affected by the shortness of the record and the unusually severe storm of 1878. Most of the curves reached their low point in the decade, 1890-1900, and, since then, have shown an irregular upward trend which may still be in progress, although the curves have been reasonably stable since 1913. If extended each year, a set of curves of this kind can be kept up to date with little labor, and will give prompt notice of any great changes in the data, which would require a revision of the rainfall curve.

In 1910, one would have said that a reliable 5-year rainfall curve could be obtained from a 42-year record; yet, if such a curve had been computed in that year, it would have been badly in need of revision by 1913, as changes in the data were nearly as great as the difference between the author's 5-year and 10-year curves. Fig. 8 shows that the author's curves, computed from the 1930 data, are still essentially correct, as changes in the data during the last three years have been small.

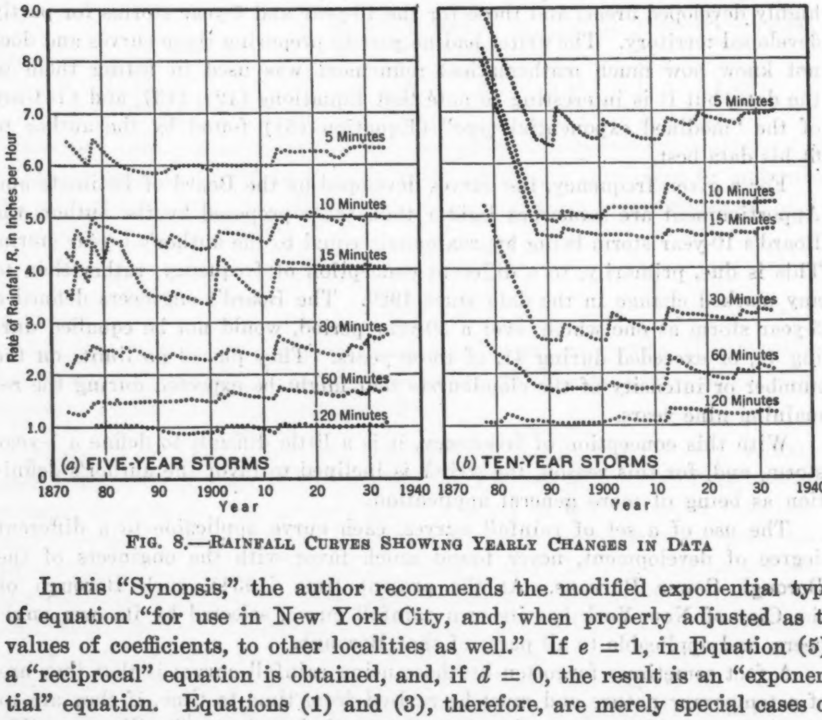

FIG. 8.—RAINFALL CURVES SHOWING YEARLY CHANGES IN DATA

In his "Synopsis," the author recommends the modified exponential type of equation "for use in New York City, and, when properly adjusted as to values of coefficients, to other localities as well." If $e = 1$, in Equation (5), a "reciprocal" equation is obtained, and, if $d = 0$, the result is an "exponential" equation. Equations (1) and (3), therefore, are merely special cases of the general type embodied in Equation (5). Of course, this type of equation can be adapted to a considerable range of data, but there is nothing in Mr. Bleich's paper to show that the author has ever attempted to fit his equations to rainfall data outside New York City. It is conceivable that rainfall data for some cities might plot in a form similar to the data for the author's 25-year or 50-year storms. The equations for these storms do not fit the data very closely, and it is probable that, for data of this character, a more complicated equation would be required. Mr. Bleich's recommendation, therefore, should be accepted with some reservations.

The author has proposed a set of rainfall curves for New York City. It is not entirely clear whether he means Greater New York, or merely the Borough of Manhattan. If the former, there is some question whether rainfall curves deduced from Central Park records would not be too severe for some of the

outlying sections. It is a peculiar fact, but local conditions seem to exist, that make some parts of the city wetter than others.

A Committee on Sewer Capacity, authorized by the Board of Estimate and Apportionment of the City of New York in 1932, secured rainfall records on an experimental area in the Borough of Brooklyn from 1925 to 1929 and on an area in the Borough of Manhattan from 1927 to 1931. Records were taken continuously from May to November of each year, using three rain gauges on each area.

The Manhattan area is in the Grand Central section of the Borough, 1 mile south of Central Park, and extends from about the middle of the Island to the East River. The Brooklyn area is in the East New York section of the Borough, 7 miles southeast of the Manhattan area and 4½ miles from the East River. In studying the records obtained on the two areas, the writer was impressed by the relatively greater intensity of the storms gauged on the Manhattan area, and, to obtain some concrete evidence on this point, made two comparative tabulations.

TABLE 12.—COMPARISON OF AVERAGE RAINFALL INTENSITIES, IN INCHES PER HOUR

Year	Location	Number of storms	Duration, in Minutes: 5	10	15	20	30	60	120
(a) Brooklyn and Manhattan Experimental Areas									
1927	Brooklyn	9	1.54	1.27	....	0.97	0.85	....	.....
	Manhattan	6	2.91	2.52	....	1.90	1.56	....	.....
1928	Brooklyn	13	1.32	1.08	....	0.85	0.72	....	.....
	Manhattan	16	2.77	2.17	....	1.42	1.07	....	.....
1929	Brooklyn	7	1.25	0.94	....	0.73	0.56	....	.....
	Manhattan	7	2.38	1.91	....	1.32	0.95	....	.....
1927–29	Brooklyn	29	1.37	1.10	....	0.85	0.72	....	.....
	Manhattan	29	2.70	2.18	....	1.50	1.14	....	.....
1927–29	Ratio: $\frac{\text{Brooklyn}}{\text{Manhattan}}$		0.51	0.50	....	0.57	0.63	....	.....
(b) Brooklyn Gauge No. 3, and Central Park Gauge									
1925	Gauge No. 3	2	1.98	1.54	1.13	....	0.64	0.40	0.23
	Central Park	....	3.00	2.58	2.20	....	1.22	0.70	0.38
1926	Gauge No. 3	6	2.98	2.36	1.82	....	1.29	0.94	0.64
	Central Park	....	2.58	2.09	1.69	....	1.15	0.87	0.56
1927	Gauge No. 3	7	2.15	1.61	1.35	....	1.00	0.71	0.48
	Central Park	....	2.42	1.87	1.66	....	1.23	0.80	0.53
1928	Gauge No. 3	6	1.65	1.35	1.20	....	1.05	0.82	0.54
	Central Park	....	3.30	2.06	1.71	....	1.09	0.66	0.43
1929	Gauge No. 3	2	1.05	0.80	0.71	....	0.52	0.69	0.42
	Central Park	....	2.34	1.74	1.52	....	0.93	0.54	0.36
1925–29	Gauge No. 3	23	2.13	1.66	1.36	....	1.02	0.77	0.51
	Central Park	....	2.73	2.03	1.71	....	1.15	0.76	0.49
1925–29	Ratio: $\frac{\text{Gauge No. 3}}{\text{Central Park}}$		0.78	0.82	0.80	....	0.89	1.01	1.04

Table 12(*a*) gives the average yearly intensities for different durations in periods when gauging was in progress on both areas. Besides storms gauged on both areas the table includes a number of storms local to Brooklyn and others local to Manhattan. The total number of storms (29) happens to be the same on each area, but the yearly distribution varies. Many light storms, which would have no bearing on sewer design, were not analyzed and, therefore, are not included in Table 12(*a*). The average intensities for the entire period show that, for durations up to 30 min, the Manhattan storms had nearly twice the intensity of the Brooklyn storms.

Table 12(*b*) gives the average yearly intensities for different durations of identical storms gauged at Central Park and at the Committee's Rain Gauge No. 3—one of the Brooklyn gauges, located 8½ miles southeast of Central Park and 5 miles from the East River. Storms local to Brooklyn, or local to Manhattan, are excluded, and Table 12(*b*), therefore, includes a considerable number of "easterly" storms, which were more likely to be gauged at both points. Table 12(*b*) includes the storm of September 6, 1926, which was of moderate intensity at Central Park, but at Gauge No. 3 was a phenomenal double-peaked storm, which, for the 120-min duration, exceeded anything ever gauged at Central Park. The 5-year averages show lighter Brooklyn intensities up to a duration of 30 min and nearly equal intensities for longer durations.

Table 12 indicates that rainfall curves for Manhattan, and for interior parts of Brooklyn, might have somewhat different shapes. The curves would be identical for the 60-min and 120-min durations, but the Brooklyn curve would have lighter intensities for the shorter durations. Of course, the table does not cover a long enough period to make the evidence of much value, but it is of some significance that, in Table 12(*a*), the lighter Brooklyn intensities are in evidence during each of the three years considered and for all four durations. The writer is willing to hazard a guess that the heavier Manhattan storms are caused by the proximity of the Hudson and East Rivers.

To summarize the viewpoint herein expressed: Rainfall curves with frequencies of from 1 year to 10 years, represented by the author's modified exponential equations, are endorsed as being close approximations of existing Central Park rainfall data. It should be recognized, however, that these curves are of a temporary nature and may need revision at any time.

Table 12 suggests that these curves may be too severe for the design of branch sewer systems in interior sections of the Borough of Brooklyn and similarly located territory in the Borough of Queens; but, unfortunately, there are no long-term rainfall records in this territory to form the basis for a rainfall curve. It is probable that branch sewer systems in this territory, designed by the author's curves, would have a large factor of safety.

While the modified exponential type of equation, doubtless, can be adapted to rainfall data in many cities, as previously stated there is nothing in the paper to show that it is of universal application. The writer has been helped in his study of this paper by the clear and concise manner in which the subject-matter has been presented.

JOSE GARCIA MONTES, JR.,[19] ESQ. (by letter)[19a].—The writer ventures to suggest that the part of Mr. Bleich's analysis, assuming formulas of the type of Equation (1), could have been improved by plotting on semi-hyperbolic paper.[20] When so plotted the data in Table 4, however, indicate that the intensities from different storms do not follow the law defined by Equation (1); nor, in fact, do corresponding intensities for a single storm follow that law. If there is any general law that applies to these data, the writer believes that it will conform more closely to a duration type of curve than to any other type so far proposed.

The consolidation of records of different types of storms according to present general practice in analyzing frequency of rainfall intensities, and the practice of assuming intensities corresponding to different types, as homogeneous records, is not justified. From a practical standpoint when it is desired to determine exactly the frequency of occurrence of given intensities in different time elements, such consolidation may perhaps be permissible; but what the writer considers unjustified is the practice of representing the interrelationship of various factors by means of mathematical equations, because the best of mathematical formulas can be only gross averages of the actual relationship. The fact that data plotted on arithmetical paper appear to follow the equations that are actually assumed for the particular case, is due probably to the effect of averaging intensities from different records, many of which, individually, might conform to quite different types of curves.

Judged as an empirical equation, Mr. Bleich's formula should be valuable to engineers practicing in and around New York City, who can make use of the data in designing storm sewers more intelligently. Based on local records covering a period of 61 years, the application of such an analysis is certainly justified. Moreover, this record is now of sufficient length to permit a detailed analysis of original values to learn, if possible, the true laws that underlie rainfall duration relations, if such laws exist. The classification of these data according to types and an independent analysis of data that fall in each class may possibly disclose interesting truths that have universal significance.

In analyzing such data statistically, the fact that it is not homogeneous is important. The rainfall probabilities applying to short duration periods must not be studied in the same class as those segregated in longer time units. The two classes of data follow different laws, those for very short time periods tending to be more stable than those for longer periods. If frequencies of the different classes of data were approximately the same, there would be some justification for consolidating the records. The frequencies are not the same, however; for example, in tropical countries the electric storm is by far the most frequent and, therefore, these control the data pertaining to storms of short duration.

Generally, storms are assumed to fall in two broad classes: (*a*) High-pressure storms including electric storms; and (*b*) cyclones. In the temperate

[19] Havana, Cuba.

[19a] Received by the Secretary March 29, 1934.

[20] *Transactions*, Am. Soc. C. E., Vol. XXXIV (1921), p. 211.

zones the first type yields most of the high rainfall intensities—up to a duration of 2 hr. In the tropics, on the other hand, at least near the Caribbean Sea, the most severe storms are mostly hurricanes or storms in the cyclone class. The latter yield the highest intensities for any duration. If now a record containing both types of intense storms is consolidated, the extreme items, such as those occurring once or twice in 50 years, would be automatically weighted to appear in the same order as storms of 15-year or 20-year occurrence. In the tropics, electric storms of a genuine catastrophic nature occur as rarely as two or three times in 50 years, whereas hurricanes and cyclones occur on the average of eight to ten times in 50 years. Consequently, the intensities and the frequency of occurrence, as well as the duration of these two classes of storms, cannot very well be consolidated. They constitute heterogeneous data that should not be analyzed in one group.

While the writer is not familiar with conditions in the United States from personal experience, his studies lead him to believe that heterogeneity of data may influence the result of analyzing intensities from records that include severe electric storms as well as storms of a cyclonic nature. A multiple electric storm (that is, two or more storms in rapid succession, or that overlap each other), yields a rainfall intensity for periods of 3 hr, or more, that is not at all comparable, by means of probability analyses, with normal cyclonic storms analyzed in the same way. As in the case of comparing West Indian hurricanes with severe electric storms, the exceptionally severe rainfall that accompanies a cyclone is less than the total rainfall produced by a recurring electric storm of 2 to 6 hr duration. In the North, the cyclonic type of storm may yield severe rainfall intensities for periods greater than 6 or 8 hr.

It would appear, then, that rainfall durations between 5 min and 6 hr, and especially between 15 min and 6 hr, are markedly inconsistent when analyzed by probability methods and are even more inconsistent when analyzed mathematically, for the reason that the record contains data of basically different characteristics. A careful classification of the data according to the class in which it belongs will show the trend much clearer, but as yet no definite law can be stated with confidence. Probably some obscure disturbing factors affect the results. Statistical analysis of data carefully segregated for the purpose in view will probably yield the kind of information that can be generalized.

A detailed record of rainfall intensities covering a period of 61 years is unique. Some of the information possible to be gleaned from it, particularly that applicable to New York City, is ably described in this valuable paper by Mr. Bleich; but information of much greater importance can be gained by a thorough classification analysis. In its entirety, information available as to rainfall intensities is quite meager. It is judged that the City of New York can well afford such a detailed study, or at least the expense of publishing the complete data available to date, thus affording an opportunity for independent investigators to extend the analysis further. Other records are available in

the United States, which cover periods of nearly 50 years, and a study of two such records, in conjunction with that of Central Park described in this paper, should prove extremely valuable in a correlation analysis.

Rainfall intensity is important to agriculture, as one of the underlying factors affecting water losses. It is also of major importance in determining the yield of water-sheds for water supply and irrigation. Because of the great labor involved in analyzing intensity data, few efforts have been made, except in the sewerage field. With the data now accummulated, it is time to stress the necessity of such analysis in other fields.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ANALYSIS OF SHEET-PILE BULKHEADS

Discussion

BY JACOB FELD, ASSOC. M. AM. SOC. C. E.

JACOB FELD,[13] ASSOC. M. AM. SOC. C. E. (by letter)[13a].—Because of the omission of a list of symbols and definitions, this paper is difficult to read and, therefore, much of its value will never be brought to the attention of many engineers who would profit by a careful study of Mr. Baumann's methods of analysis. Furthermore, it would be easier to follow the author's mathematics if the source of the equations were given.

The object of the paper is threefold—and the writer's opinion of each section, in summary, is as follows:

1.—The description of tests made to determine effective passive resistance of sand and of interlock efficiency of sheet-piling is so detailed, for a clear description, that even a re-reading leaves one without a picture of what was intended or what was determined. The results obtained are open to serious criticism, as described more fully hereafter.

2.—The suggestion of a possible stress distribution in the passive prism (or soil-volume resisting pressure) in as far as it applies to the particular problem of the failure of certain sheet-piling at Long Beach, Calif., is based on the results of the tests, and is so full of additional assumptions, that it is not convincing. In so far as it deals with the general problem, keeping in mind the assumptions made, it is a real contribution to the study of soil mechanics.

3.—The new theory of the stability of bulkheads is an ingenious mathematical analysis.

A test is described in which deformations of seven interlocked sheet-piles, embedded 12 ft in sand and acted upon by a hydrostratic pressure of 15 ft of water, were measured. The set of sheets is described as free to deflect without restraint from the side walls of the bin holding the water. The walls of

NOTE.—The paper by Paul Baumann, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[13] Cons. Engr., New York, N. Y.

[13a] Received by the Secretary April 6, 1934.

the bin also consisted of sheet-piles driven to the same depth as the test sheets. The purpose of the test was to simulate the condition of a sheet-pile bulkhead which had failed.

There can be no question as to the total value of the acting pressure, on all the sheets. However, the proximity of the rear of the bin to the tested face (about 5 ft), seriously affected the capacity of the soil to resist pressure. This was most serious near the side walls, where these walls, also acted upon by the water pressure, caused resisting stresses in the soil which was supposed to sustain the test piles. As a result, the end piles were forced to throw part of their load toward the center piles which, in addition, were not provided with a full passive prism. An inspection of Fig. 6 showing permanent deformation in the test sheets certainly bears out the foregoing statements. The loads taken by the various sheets were not equal. The analyst makes the assumption that each sheet sustained the same acting pressure. When a permanent deformation of about 2.5 in. remains after a measured maximum deformation of 4.16 in., one wonders what right the author has to use the elastic stress relation to convert strains into stress.

Some data for the evaluation of the parameters, unit weight, and internal friction required for the interpretation, are listed. However, the following assumptions are made in converting the measured deflections of the sheet-piling to a diagram showing the amount and sign of the pressures acting on the sheet-pile for its full height (Fig. 5):

(*a*) The angle of internal friction equals the angle of natural slope. Some difficulty was found in measuring even the angle of natural slope in the immersed or saturated state, and no attempt was made to determine the coefficient of internal friction.

(*b*) The weight of the average sand was 85 lb per cu ft with 15 to 20% moisture by weight. The great range of values in Table 1 shows a non-homogeneous fill.

(*c*) The angle of internal friction was assumed to be 30° under water.

(*d*) The active pressure of the soil inside the sheet-pile enclosure was assumed to vary parabolically; the active pressure is assumed to act at the same time as passive resistance (see Fig. 5). If the soil was saturated throughout the test, there is no question as to the amount or variation of active pressure, except as the bin action of the enclosure affects the pressure of the sand; the water pressure is definite. Active pressure and passive resistance may act on the same sheet on opposite sides, but not on the same side. The latter is equivalent to assuming motion of the sheet in two opposite directions at the same time.

(*e*) No attempt was made to evaluate the effect of water seeping from the bottom of the test bin.

(*f*) Deflections were computed for several values of a multiplier inserted into the Coulomb formula for passive resistance, and the curves were plotted. A value of 1.68 for the multiplier is arrived at, because that deflection curve, plotted for the full depth of the sheet-pile (27 ft), is considered nearest the deflection curve plotted from the observed deflections. Since the measured

deflections were for the top 15 ft only, and the amount of deflections measured is about twice that computed, the real reason for this choice of factor is far from self-evident.

(*g*) Following a discussion of different assumptions as to the maximum value of passive resistance in front of, and in back of, the test wall, and the effect of wall friction and wall deformation on these values, there appears the statement (see, "Method of Graphic Analysis"): "As to the test under consideration, the passive resistance behind the wall was assumed to be 70% or less of the passive resistance in front, because the surcharge was water." Since there was no surcharge in front, why the reduction? A reduction was required for the effect of the rear wall, not for lack of earth surcharge.

The results of the test analysis are next applied to a proof that the sheet-piling bulkhead which failed should have failed. In addition to the foregoing assumptions, it should be pointed out that the bulkhead had a concrete relieving platform resting on timber piles (assumed by the author to take one-third the active earth pressure); and that the "effectiveness ratio" or multiplier of the Coulomb formula equals 2. A method of trial wedges is used to determine the wedge of minimum resistance, assuming such wedge to start at the bottom of the sheet-piling (Fig. 7). If this assumption is true, there cannot be a balancing force at the heel of the pile, and equilibrium cannot exist. The writer cannot see any justification for applying the results of the test to the analysis of the bulkhead; nor is the computed maximum deflection of 4.13 in. convincing. With a yield point of 45 782 lb per sq in., and a maximum stress of 30 000 lb per sq in., there should not be a large amount of permanent set; and, yet, the author states that the shape of the sheets pulled at the bulkhead that had failed agreed with that of the test sheets (Fig. 6) and these had a permanent set of 2.5 in. after a maximum deflection of 4.16 in.

The "Analytical Determination of Interlock Efficiency" is based on approximating a curve the equation of which is known, to the plotted shape of the test piles. The results are subject to the same criticism as that which applies to the test. In addition, the method used is only applicable to deformations within elastic limits, and the measured deformations certainly exceeded that limit.

The suggested theory of stability is a good form of attack on the problem. However, the author fails to point out that it only applies within very small limits, practically within elastic deformations of the soil structure. In stating that: "In the foregoing, * * * earth was treated like a fluid of smaller or greater unit weight than earth for active and passive resistance, respectively," the author falls into an improper use of the word "fluid." A fundamental law of fluids can be re-worded into the requirement that at all depths active pressure is equal and opposite to the passive resistance to pressure.

Before accepting the author's formulas quantitatively, the assumptions must be further checked by experiment. Equation (16) involves the assumption that the passive resistance to pressure varies inversely with an indefinite power of the depth and with the square root of the deflection. Equation (18) shows a relation from which the unknowns may be determined if the lateral

resistance at various movements can be measured. However, the value at zero movement must first be measured, this being called the active pressure. The writer is convinced that at zero movement there is no method for measuring the pressure, either active or passive.

Before using the results obtained by Franzius, it should be carefully noted that the wall used in his tests was moved laterally against the soil, that the maximum pressure did not occur at the beginning of movement, but only after the body of the fill was crushed. Such complete lateral movement does not occur in the case of sheet-piling. When the wall is pushed into the fill, the total resistance measured is the sum of (*a*) the passive resistance of the earth to pressure; (*b*) the resistance to the crushing of adjacent grains; and (*c*) the resistance caused by physical displacement.

If no friction exists along the wall (a hypothetical case) the last item disappears, because the soil mass merely adjusts itself by moving upward, just as a liquid would. If friction does exist, a resistance greater than the Coulomb formula would indicate, is measured. Incidentally, the Coulomb formula does not involve the assumption that the amount of movement is large enough to induce any resistance except the passive resistance of the weight of that mass in back of the wall which will give the minimum resistance.

For the benefit of those who do not have ready access to the original papers by Franzius, a summary of results obtained have been made available[14] by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E. The conclusions listed in Mr. Freeman's book do not agree with some of the assumptions made in this paper.

The writer cannot see how Equation (25) is derived. For deformations within elastic bounds, a term containing the tenth root of the deformation certainly cannot be very considerable in magnitude.

Following Equation (33) the author states that his analysis shows a decrease in the coefficients of internal friction and of friction between sand and wall, with an increase in depth and an increase in wall movement, and also refers to the writer's experiments,[15] for qualitative agreement. Before completing this discussion, the writer tried to find such agreement in his own report and found just the opposite. In his experimental work he found no appreciable variation in either coefficient with either depth or amount of wall movement. Experimental work on the variation of the coefficient of internal friction with applied load has consistently shown no relationship.

The writer agrees that the friction along the wall is responsible for most of the complexities in the phenomenon of passive resistance (see text following Equation (42)). A clearer picture can perhaps be drawn by considering the resistance prism as being acted upon by a drag equal to the friction along the wall. This drag is dissipated through the fill, and causes a curvature in the surface of the rupture.

In view of the foregoing remarks, the writer cannot agree with the conclusions of the paper.

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[14] "Hydraulic Laboratory Practice," by John R. Freeman, pp. 599-603.
[15] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.

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DISCUSSIONS

A GENERALIZED DEFLECTION THEORY FOR SUSPENSION BRIDGES

Discussion

BY E. L. PAVLO, ESQ.

E. L. PAVLO,[5] ESQ. (by letter)[5a].—The Deflection Theory for suspension bridges with stiffening trusses hinged at the towers became generally known to the profession through its practical development and application to the design of the Manhattan Bridge by L. S. Moisseiff, M. Am. Soc. C. E., in 1904. Since that time several bridges have been built with the stiffening trusses continuous over the towers because of their estimated superior rigidity and economy over the hinged type. Because there was no exact theory for the analysis of stresses in continuous trusses, the design of these bridges has necessarily been approximate and the advantages of the continuous type could not have been definitely ascertained.

Mr. Steinman eliminates this deficiency and also extends the use of the Deflection Theory to the cases of multiple suspension bridges with or without tie cables. His paper is a notable scientific contribution to the theory of suspension bridges. The analysis of the stresses in the continuous stiffening trusses is a difficult and involved mathematical problem. Aside from the theoretical complexity it presents an algebraic difficulty in transforming and condensing the extremely complex formulas into workable expressions. The paper demonstrates the remarkable ingenuity of the author in overcoming these difficulties. Only those who have attempted the same task can fully appreciate the time and energy required.

In 1932 the writer undertook to verify, by an independent method, the conclusions of Mr. Steinman's original paper, and to obtain some additional data for the economic comparison of the hinged and continuous types of stiffening trusses. For this purpose the writer developed an analysis quite similar to that of the present paper, which is likewise based on the equality of the slopes to the elastic curve at the towers. Being primarily interested in the practical applications of the theory, rather than in developing a finished treatise on the subject, the writer strove to correlate his derivation with the already existing theory for the hinged trusses. In doing so he devised an easy means of

NOTE.—The paper by D. B. Steinman, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[5] White Plains, N. Y.

[5a] Received by the Secretary April 18, 1934.

obtaining equations for continuous trusses from the existing equations[6] derived for the hinged condition, simply by adding certain constant quantities, say, Δ_1, Δ_2, and β to the existing equations for C_1, C_2, and H_1, respectively. This procedure will be particularly useful to those already familiar with the Deflection Theory as applied to the hinged stiffening trusses.

In evaluating the constants of integration, C_1 and C_2, the only difference between the derivations for the hinged and continuous conditions is that in the case of hinged trusses M is equated to zero for $x = 0$ and $x = l$; whereas, in the case of continuous trusses M is equated to T_1 and T_2. This change in the boundary conditions creates additional constant terms, representing the effect of continuity, in the expressions of C_1 and C_2, for various loading conditions. These terms are:

In the main span,

$$\Delta_1 = \frac{T_1\, e^{-cl} - T_2}{H\,(e^{cl} - e^{-cl})} \qquad (51)$$

and,

$$\Delta_2 = -\frac{T_1}{H} \qquad (52)$$

In the side spans,

$$\Delta_1 = -\frac{T_{1,2}}{H\,(e^{c_1 l_1} - e^{-c_1 l_1})} \qquad (53)$$

and,

$$\Delta_2 = 0 \qquad (54)$$

All the formulas for the integration constants may thus be easily written from the formulas of the hinged case, given in textbooks, by simply adding Equation (51) or Equation (53) to C_1 and Equation (52) or Equation (54) to C_2.

For symmetrical loading cases $T_1 = T_2$, and Equation (51) becomes,

$$\Delta_1 = -\frac{T_1}{H\,(e^{cl} + 1)} \qquad (55)$$

To obtain the formulas of H for various loading conditions it is sufficient to add the term,

$$\beta = (T_1 + T_2)\,\frac{F}{2} \qquad (56)$$

to the numerators of the known formulas for the hinged trusses, the denominator being left unchanged. The use of H-formulas containing temperature terms both in the numerator and denominator is not permissible for the present purpose and will introduce an error. The general form of the H-formula is given by Equation (24). The writer finds it easier to use this type of equation than Equation (27) in which the term $(T_1 + T_2)$ is partly eliminated at the expense of complicating the denominator, for the reason that $(T_1 + T_2)$ must be found in any case for use in the equations of C_1 and C_2. In case of computations for trial values of H the writer also finds it more advantageous to use Equation (24).

[6] "A Practical Treatise on Suspension Bridges," by D. B. Steinman, Second Edition John Wiley, 1929.

The continuity at the towers introduces the additional two unknowns, T_1 and T_2, which are treated as the known quantities although they vary with each loading condition. At the same time the continuity of the elastic curve at the towers provides the additional two equations for the two unknowns it created. These equations are obtained by equating the slopes to the elastic curves on each side of the towers.

In the case of a bridge fully loaded with the load, p, in the center span and the load, p_1, in the side spans the application of the slope equations yields,

$$T_1 = T_2 = H \frac{\left[\frac{d}{c}\left(\frac{p}{H} - \frac{1}{r}\right) + \frac{d_1}{c_1}\left(\frac{p_1}{H} - \frac{1}{r_1}\right) + 4\,(n + n_1) - \frac{(p\,l + p_1\,l_1)}{2\,H}\right]}{\left[cd + \frac{c_1}{2}\left(d_1 + \frac{1}{d_1}\right) - \frac{1}{l_1}\right]} \quad \ldots (57)$$

In the case of the center span fully loaded and no load on the side spans, place $p_1 = 0$ in Equation (57). In the case of side spans fully loaded with the load, p_1, and no load on the center span, place $p = 0$ in Equation (57).

In the case of partial or full unsymmetrical loadings, the general algebraic equations for T_1 and T_2 became too complicated for practical use and it is more advantageous in these cases to write out the expressions for $(T_1 + T_2)$ and $(T_1 - T_2)$ by substituting the appropriate contributions, B_1 and B_2 (tabulated in Article 9 of the paper) into Equations (19*a*), (19*b*), (35*a*), and (35*b*), and to solve numerically the resulting two equations for T_1 and T_2.

In the practical application of the Deflection Theory for continuous trusses, the problem of finding the moments of inertia, I and I_1, requires further elucidation. The writer quite agrees that the effect of the variation of I within a span may be ignored; the difficulty, however, lies in the fact that it is impossible to find correct single values of the moments of inertia to apply for all loading conditions as in the case of hinged stiffening trusses. The deflection theory is based upon the deflections of the stiffening member as expressed by the beam flexure equation, $\frac{d^2 n}{dx^2} = -\frac{M}{EI}$.

Inasmuch as in most cases the stiffening member is a truss and not a beam, it is necessary, for the application of the theory, to find an equivalent beam such that its deflection characteristics would approximate those of a given truss. For the two-hinged trusses this equivalent beam is found by equating computed deflections of the truss under certain loadings to the corresponding beam deflection formulas and solving for I. The average of the values of I for various loading conditions represents the so-called "equivalent moment of inertia" of the truss. It is found that for the two-hinged trusses the individual values of I for various loading conditions differ but little from each other; and, therefore, it is assumed that their average value can apply for all the loading cases.

In case of a three-span continuous truss of symmetrical design, the determination of equivalent values of I for the center span and I_1 for the side spans requires finding deflections in a statically indeterminate structure. After finding these deflections in one side span and in one center span under a

certain loading, they are equated to the corresponding equations of the three-span beam deflections which contain both I and I_1. In this way two equations are obtained which are solved simultaneously for I and I_1. In this manner the writer analyzed a normal design of a three-span continuous truss for various full-span loadings and found that the individual values of equivalent moments of inertia differed considerably for some loadings.

Since equivalent moments of inertia that can apply for all loading cases can not be found, it is necessary, in the practical application of the deflection theory, to prepare a set of moments of inertia for the several representative loading conditions. The problem is further complicated by the fact that the representative loading conditions can only be known tentatively since the actual loading on the truss is variable on account of the variable suspender pull.

The method of finding equivalent moments of inertia by multiplying the average chord area by the square of one-half the height of the truss should not be used in the case of continuous stiffening trusses. In the case of two-hinged trusses this method is sometimes used since it gives values close enough to those obtained by equating the deflections. In the case of continuous trusses it has no such justification, and the deflection method of finding one equivalent beam is the only rational method to be used.

The example of finding the stresses and deflections in a three-span continuous suspension bridge is very instructive. The economic comparison of hinged and continuous trusses is also of great interest to the designing engineer. The writer is of the opinion that specifications for suspension bridges should contain rigidity requirements in terms of maximum permissible change in grade instead of deflection ratios. On that basis the comparison of the hinged and continuous trusses would be more advantageous for the latter. The writer agrees with Mr. Steinman that a 1 000-ft span marks the upper limit of the economic advantages of continuous suspension bridges in so far as the saving of material in trusses is concerned. For long spans the continuity is comparatively unimportant since it affects relatively small lengths of span immediately adjacent to the towers. In long-span suspension bridges the rigidity is derived through the preponderance of the dead load over the live load. If the ratio of dead to live load is sufficiently high the stiffening trusses may be dispensed with entirely as was done in the design of the George Washington Bridge. In shorter spans, the continuity assumes a far greater importance.

There is a misconception on the part of many engineers that suspension bridges can apply economically only for long-span crossings. By building the short-span suspension bridges with trusses continuous over the towers the requisite rigidity can be attained and a saving in material effected. Suspension bridges can thus penetrate economically into the domain hitherto believed to be reserved for other types of bridge construction. The multiple, short-span suspension bridges also present an excellent economic solution for relatively shallow crossings. The present paper affords a means of designing such suspension bridges rationally, and therein lies its great importance.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PRACTICAL RIVER LABORATORY HYDRAULICS

Discussion

BY J. B. EGIAZAROFF, ASSOC. M. AM. SOC. C. E.

J. B. EGIAZAROFF,[38] ASSOC. M. AM. SOC. C. E. (by letter)[38a].—A number of very interesting and important questions are developed in this excellent paper. Its special merit lies in the fact that its author has brought to a conclusion the answer to practical questions that formerly were explained only in a general way.

Two aspects of the problem should be emphasized: (1) The value of the criterion for turbulent flow in models; and (2) conditions to be met in devising fluvial models to accommodate a movable bed load.

(1) The Value of the Criterion for Turbulent Flow.—The author has shown that, for the Reynolds' criterion as proposed by Professor H. Krey, $V D > 0.075$, can be substituted by $V D > 0.02$, even for a smooth concrete-lined bed.

Consequently, for a model with a movable bed load (sand), this condition will give results even more favorable, on account of the greater roughness of the bed. Therefore, it becomes possible to experiment with non-distorted models which, according to Professor Krey, it was necessary, formerly, to distort.

Of great influence in the choice of the scale of a model is that part of its cross-section, that is in active motion. The entire cross-section of a river does not take part in this motion; in a part of the channel (especially in cases when water flow is retarded by a dam) the water has a mild eddying motion. The application of the Reynolds' criterion to the entire cross-section, including those parts not actively involved in the motion of the stream, seems to be incorrect.

(2) Conditions to Be Met in Designing River Models.—Of exceptional interest is the question of similitude in models designed to accommodate a

NOTE.—The paper by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1934, by Messrs. Lorenz G. Straub, Paul W. Thompson, Ralph W. Powell, K. D. Nichols, and Frank W. Edwards; March, 1934, by Messrs. I. H. Patty, Charles S. Bennett, and Kenneth C. Reynolds; and April, 1934, by Messrs. V. V. Tchikoff, Samuel Shulits, and Charles D. Curran.

[38] Prof. and Director, Hydro-Elec. Laboratory, State Scientific Research Inst. on Hydrotechnics, Leningrad, Union of Socialistic Soviet Republics.

[38a] Received by the Secretary February 24, 1934.

movable bed. This paper and others emanating from the U. S. Waterways Laboratory, indicate clearly how to choose sand as to size and quantities, and the corresponding rate of flow for any experiment. Hans Kramer, Assoc. M. Am. Soc. C. E., particularly, has contributed valuable information in this field.[39]

One serious difficulty remains, however, the task of obtaining sufficient data as to the movement of the bed load in Nature (the prototype), so as to create the same condiitons in a laboratory flume. This problem is especially serious in the forebays of hydro-electric plants where, after sedimentation, particles are put in motion, which could not be considered as present in the normal bed load of the stream.

From 1931 to 1933 experiments were performed in the Hydro-Electric Laboratory in Leningrad,[40] on three models of head-works with intakes. Observations were made of the behavior of a movable bed load in a river with a steep slope, and in a laboratory flume. All these experiments showed, that when the size of sand particles chosen for the model was limited to 0.5 mm to 2 mm and when the average effective diameter did not exceed 1.5 mm, the new bed of the river, in the forebay, assumed a slope that was practically independent of the size of the sand particles.

This new bed slope, however, was greatly influenced by the ratio of the quantity of sand to the quantity of water that passed through the model at the same time. The results thus obtained make easier the selection of model "geschiebe" for torrential streams retarded by dams, than would be necessary for freely flowing streams, with relatively flat slopes and transporting a finer silt.

Naturally, comparisons of observations on a model and prototype are of the utmost importance. Such a comparison was made in 1933 at the Hydro-Electric Laboratory for the hydro-electric plant at Zemo-Avtchally, U.S.S.R. The scale ratio of the model (non-distorted) was 1:100. The forebay of the prototype was fully silted after two years, with little change in the succeeding three years; the material was composed of fine sand in suspension, without gravel or boulders. In the model the sedimentation was entirely the result of "geschiebe" movement. Comparisons of the silted sections of prototype and model indicated a discrepancy of 6 to 8% near the head-works and intake structure, and as much as 25 to 35% farther up stream where a sharp curvature of the channel may have influenced the result.

[39] *Proceedings*, Am. Soc. C. E., April, 1934, p. 443.

[40] See, "Description of the Laboratories of the State Scientific Research Institution on Hydrotechnics in Leningrad," 1933, and *Proceedings* of the same Institute, Vol. IX and X (Engineering Societies Library).

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DISCUSSIONS

AN APPROACH TO DETERMINATE STREAM FLOW

Discussion

BY W. W. HORNER, M. AM. SOC. C. E.

W. W. HORNER,[17] M. AM. SOC. C. E. (by letter)[17a].—Inspired (as he states) by Mr. Sherman's demonstration of the "unit graph," the author has taken another valuable step in systematizing the relation of rainfall and run-off. No doubt, the distribution graph will be accepted by hydrologists as a workable mechanism.

Many students of this subject have been of the opinion that the present difficulty in establishing a rational relation between run-off and rainfall might be largely overcome if the effect of the fixed drainage-area characteristics, such as shape, slope, and channel condition, could be isolated. In another connection,[18] Mr. Bernard made a nice academic approach to the problem; in his paper, he indicates a practical means of achieving this result.

He refers to his work as an "approach to determinate stream flow"; the qualification should be kept well in mind. Apparently, he has examined a great mass of data and has subjected it to analysis by his method. To substantiate the hypothesis fully will require a study on a larger scale than could be undertaken by any one person; it is to be hoped that Mr. Bernard's effort will bring about the necessary organization.

The idea that, for rain within the chosen time unit, run-off diagrams will fall into definite patterns corresponding to salient water-shed characteristics, and that this hydrograph may be considered as independent of variations in rainfall intensity within the time unit, will undoubtedly require some qualifications, as the author suggests under "Limitations of the Method." It is possible that variations in ground condition will be found to account for nonconformities with the distribution graph; that is, as certain areas are relatively more retentive than others at certain seasons, the effect may be similar

NOTE.—The paper by Merrill M. Bernard, M. Am. Soc. C. E., was published in January, 1934, *Proceedings*. Discussion in this paper has appeared in *Proceedings*, as follows: March, 1934, by C. S. Jarvis, M. Am. Soc. C. E; and April, 1934, by LeRoy K. Sherman, M. Am. Soc. C. E.

[17] Cons. Engr., St. Louis, Mo.

[17a] Received by the Secretary April 11, 1934.

[18] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1150.

to a shift in the water-shed boundaries. In recent studies of small areas, the writer has encountered vagaries that might be accounted for in this manner.

The writer's experience with small areas leads him to feel that Mr. Bernard's viewpoint as to "Coefficients" is too optimistic. For these small areas a synthetic treatment of the rainfall resulted in the development of "100% run-off curves," corresponding to Mr. Bernard's pluviagraphs. The method was adjusted so that these curves were quite similar to the measured hydrographs. A simple relation between the ordinates of the two diagrams was anticipated, but the results were disappointing, the ratios covering a wide range of values without appreciable seasonal significance.

It is doubtful whether, even within a particular storm period, a straight-line variation between rainfall and run-off will be found; probably the relation will be more complex, and the seasonal fluctuation will be difficult to index. In passing, the writer suggests that the coefficient of retention be a "coefficient of non-retention"; it is very like the old "percentage of run-off."

The suggested preparation of frequency pluviagraphs needs further study; it does not seem to follow that, because a value of C equal to unity occurs over a year, "the pluviagraph of a storm of known frequency may be taken as a hydrograph that will be reached or exceeded with that frequency"; for this to be true, it would seem to be necessary that the unit, C, persist throughout the year. The writer's studies indicate that high run-off values are generally the result of a combination of high values of C with not excessive precipitation intensities, but sometimes from the alternate combination. He is doubtful whether any satisfactorily simple plan can be devised for studying probabilities of the concurrence of particular values.

Mr. Bernard's method appears to be most valuable, with the present lack of knowledge of coefficients, as a suggestion for analyzing existing information on a large scale. If such analysis should produce regular patterns and consistent factors, it may become the plan of construction for synthetic hydrographs.

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DISCUSSIONS

DISCHARGE FORMULA AND TABLES FOR SHARP-CRESTED SUPPRESSED WEIRS

Discussion

By Jasper O. Draffin, M. Am. Soc. C. E.

Jasper O. Draffin,[8] M. Am. Soc. C. E. (by letter)[8a].—The view is held by some hydraulic engineers that a weir is not an acceptable device for measuring the flow of water, partly because no formula has been found which will give with sufficient accuracy the discharge for all heads and weir heights. In this paper the author derives a formula which he claims is more exact and easier to use than other existing formulas and which agrees with experiments. This still leaves untouched the question of whether or not a given sharp-crested weir will continue to discharge at the same rate after it has been in use for several years.

The formula proposed by Mr. Cline is more complex than the Rehbock formula, or that proposed by Schoder, but it may be used with ease when tables or curves have been constructed, as they have been by the author.

The Bazin and Francis formulas do not give results which agree with the Schoder and Turner experiments, except for limited ranges of head and weir height, but the results given by the Rehbock formula agree very closely with the measured quantities for heads not in excess of 2 ft and for weir heights of not more than 4 ft. Beyond these values the Rehbock formula is but little better than either the Francis or the Bazin formula and none of them agrees with the Schoder and Turner experiments. Rehbock accounts for this lack of agreement by claiming that "the data for the 5.5 and 7.5-ft weir (Series D, K, L. M, and O, Table 1) are obviously unreliable,"[7] He attributes the lack of reliability to the approach channel conditions, measurement of head, and the possibility of insufficient aeration of the jet, and shows that the data are not consistent among themselves. Schoder, however, points out that the jet was fully aerated. Rehbock also states that "sharp-crested weirs more than 4 ft in height are almost never used in practice."[7]

Note.—The paper by C. G. Cline, Esq., was published in January, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all the members for further discussion.

[8] Associate Prof., Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

[8a] Received by the Secretary February 17, 1934.

[7] "Precise Weir Measurements," *Transactions*, Am. Soc. C. E. Vol. 93 (1929), p. 1155.

The Cline formula gives results which are decidedly better than any of the other formulas when the whole range of heads and weir heights is considered. Even if the Schoder and Turner data for high heads and weir heights are somewhat erratic, the Cline formula gives a good average of the measured quantities. The fact that Equation (23) agrees so closely with the experimental quantities at the lower heads and weir heights, as well as at the higher ones, indicates that the form of the equation is correct. Of course, the weak point in it is the fact that the constants were derived to fit data which seem somewhat erratic and which might be different under different experimental conditions. However, the author does not propose the formula as a final one, but as one which may be modified as further experiments are made, and he has made clear the method which should be followed in making a revision. A series of experiments should be carefully planned and performed in which the approach channel, aeration of jet, and measurement of head will all be of an accepted standard condition. Then, if the Cline formula does not exactly fit the new data, the constants in it may be modified in the manner which he suggests.

Certainly, the author deserves much credit for his splendid approach to the problem and for the large amount of work which he has done in deriving the formula which he proposes, and in computing the tables.

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DISCUSSIONS

RENEWAL OF MITER-GATE BEARINGS, PANAMA CANAL

Discussion

By E. S. Randolph, M. Am. Soc. C. E.

E. S. Randolph,[4] M. Am. Soc. C. E. (by letter)[4a].—The subject has been so well covered in this paper that there is little comment to offer. The first, or experimental, repairs conducted under the technical supervision of the writer, were reported in 1930.[5] It is gratifying to the engineer of the original equipment that much of it was still in use in 1933, after being previously used in 1929 and 1931. An important event in the 1931 operation was the test that was run as a preliminary drill and exercise for the apparatus and crew before attempting to lift a gate. Had not this test been run, there would have been serious trouble.

The preliminary test was made with all equipment for one gate assembled and a load of structural material instead of the gate. The test load was much lighter than the gate. It was noted that the travel of the jacks was at uneven rates. Examination showed scoring of the rams and cylinders in seven of the twelve jacks; one had a groove $\frac{3}{8}$ in. wide and $\frac{3}{32}$ in. deep; furthermore, foundry sand and some metal chips and cuttings from drilling and tapping operations were found in the cylinders of the jacks. New rams of case-hardened material were made; the cylinders were thoroughly cleaned; and felt wiping washers were provided at the end of each cylinder so as to fit tightly against the rams. The improved jacks were finally assembled in the machine shop and kept in a locker until used; not the least trouble from scoring was experienced thereafter.

Another preliminary test was made by placing two jacks with rams in opposition. Yokes and links, as provided for regular operation, were used, and additional double links completed the tension members. The jacks were then loaded to 110 tons of 2 000 lb without any signs of distress.

Note.—The paper by Clinton Morse, Jun. Am. Soc. C. E., was published in January, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[4] Constr. Engr., Madden Dam, Canal Zone.

[4a] Received by the Secretary March 19, 1934.

[5] *The Military Engineer*, May-June, 1930.

While being constructed, more clearance under the gates would have facilitated their removal; and wider sills would have made special foundations for the jacks unnecessary. The gates were erected in a raised position, moved against the lock-wall, lowered over the pintle bearing, and yokes were fitted to the top. While being erected and moved, each leaf was supported on two steel falsework frames, one on each side. These frames were supported on wedges for lowering the leaves, and the wedges rested on roller nests for moving the gates to the walls. Consideration was given to re-assembling these frames for repairing the gates with jacks used instead of wedges, but this was found to be too laborious for the work intended. A description of the gates and their erection was published[6] in book form in 1916.

[6] "The Panama Canal," by George W. Goethals, M. Am. Soc. C. E., Vol. II, New York, McGraw-Hill Pub. Co., 1916.

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DISCUSSIONS

FLEXIBLE "FIRST-STORY" CONSTRUCTION FOR EARTHQUAKE RESISTANCE

Discussion

BY MESSRS. LEE H. JOHNSON, EDWARD J. BEDNARSKI, AND MERIT P. WHITE AND PAUL L. KARTZKE

LEE H. JOHNSON,[8] ESQ. (by letter)[8a].—This is a helpful and thought-provoking paper on a subject of some controversy among structural engineers to-day. It is gratifying to note that the author points out that, in determining the effects of a vibration on a structure, the period of the acceleration is just as important as the acceleration itself. It should be emphasized even more strongly that the effects of a vibratory phenomenon, upon any object cannot be determined by considering only the acceleration. This fact seems to have been obscured by an imperfect understanding of the relation between acceleration, frequency, and amplitude of a vibration. Considering the simplest of periodic motions, a simple harmonic motion of the form,

$$y = D \sin \omega t \qquad (14)$$

in which, D = maximum amplitude of vibration, and ω = a constant angular velocity $2 = \pi F$, in which F = frequency by vibration, the acceleration is given by,

$$\frac{d^2y}{dt^2} = - D\,\omega^2 \sin \omega t = - D\,F^2\,4\,\pi^2 \sin \omega t \qquad (15)$$

from which it is seen that the acceleration is a function of the amplitude, D, times the square of the frequency, F, and that it is, therefore, dependent upon both quantities. It follows from this that the acceleration varies as $D\,F^2$, and that two vibrations, differing widely in character and in their effect on a structure, may yet have the same acceleration. A vibration with a very high frequency and infinitesimal amplitude can perfectly well have the same acceleration as a vibration of relatively low frequency and appreciable amplitude, the product, $D\,F^2$, being the same in each case. However, the effect of a vibration of the former type on a building will, ordinarily, be negligible, while the effect of a vibration of the latter type, such as an earthquake, can be disastrous. If

NOTE.—The paper by Norman B. Green, Esq., was published in February, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[8] Asst., Harvard Eng. School, Harvard Univ., Cambridge, Mass.

[8a] Received by the Secretary March 20, 1934.

the frequency decreases still more to infinitesimal proportions and the amplitude becomes very great, the acceleration still remaining unchanged, a building would probably suffer no ill effects, although it would be moved a considerable distance from its position of rest. Apparently, this last case does not occur in practice.

Hence, it is necessary to know at least two of these quantities, acceleration, frequency, and amplitude, not only in order to find the effect of a vibration on a structure, but, what is still more fundamental, in order to describe the nature of the vibration itself. Although such a simple relation between these quantities, as the preceding, may not exist in the case of more complex, non-harmonic vibrations, still the quantities are related and are not independent.

The author's mathematical analysis calls for some remarks. To understand these it is well to have the fundamental concepts clearly in mind. A simple harmonic motion of amplitude, D, and frequency, F, may be represented by the motion of the projection upon a fixed line of a vector of magnitude, D, rotating at constant angular velocity, as shown in Fig. 4. In this case, $y = D \cos \omega t$.

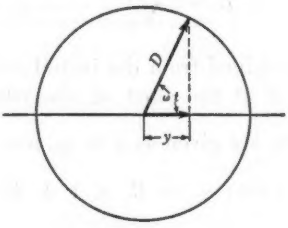

FIG. 4.

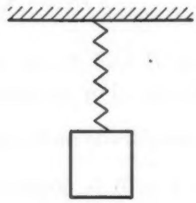

FIG. 5.

Examples of systems that give simple harmonic motion are a mass suspended on a perfectly elastic spring attached to a rigid support (Fig. 5) and a mass resting on frictionless rollers between perfectly elastic springs attached to rigid supports (Fig. 6). The masses are assumed to be non-elastic and the springs are assumed to be weightless.

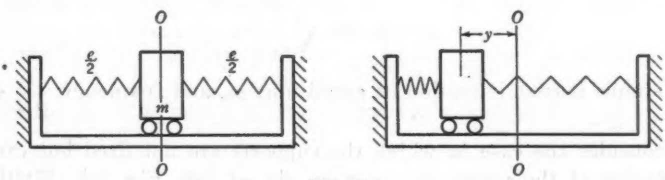

FIG. 6.

The author uses a system of the type shown in Fig. 6 as being analogous to a building supported on flexible columns with its upper stories constituting a rigid mass. The columns act as springs between the mass and the ground. Considering this system, the differential equation for the motion of the mass, m, is:

$$m \frac{d^2 y}{d t^2} = - e y \quad \text{.......................(16)}$$

which means that at any instant the inertia force of the mass must be equal and opposite to the restoring force of the springs, an application of one of the basic laws of dynamic equilibrium. Writing the equation in the form:

$$\frac{d^2y}{d\,t^2} = -\frac{e}{m}\,y \quad \text{.........................(17)}$$

is equivalent to stating, simply, that the second derivative of a function is proportional to the function itself, and since sine and cosine functions are of this type, a solution may be assumed, of the form, $y = B_1 \cos \omega t + B_2 \sin \omega t$, in which, B_1, B_2, and ω are unknown. Substituting this in the differential Equation (17):

$$-B_1\,\omega^2 \cos \omega t - B_2\,\omega^2 \sin \omega t = -\frac{e}{m}\,(B_1 \cos \omega t + B_2 \sin \omega t)$$

whence, $\omega^2 = \frac{e}{m}$, $\omega = \sqrt{\frac{e}{m}}$, and $F = \frac{1}{2\,\pi}\sqrt{\frac{e}{m}}$, and the solution becomes:

$$y\;B_1 \cos \sqrt{\frac{e}{m}}\,t + B_2 \sin \sqrt{\frac{e}{m}}\,t \quad \text{................(18)}$$

The constants, B_1 and B_2, are determined from the initial conditions of the problem considered. For example, if at the start of the vibration when $t = 0$, the initial amplitude and velocity are given as $y = y_0$, and $\frac{dy}{dt} = 0$ then, substituting for $t = 0$ in Equation (18): $y_0 = B_1 \times 1 + B_2 \times 0$, whence, $B_1 = y_0$. Now,

$$\frac{d\,y}{d\,t} = -B_1\sqrt{\frac{e}{m}}\,\sin\sqrt{\frac{e}{m}}\,t + B_2\sqrt{\frac{e}{m}}\,\cos\sqrt{\frac{e}{m}}\,t$$

and, for $t = 0$. $0 = -B_1\sqrt{\frac{e}{m}} \times 0 + B_2\sqrt{\frac{e}{m}} \times 1$; whence, $B_2 = 0$. The final solution for this case is, therefore,

$$y = y_0 \cos \sqrt{\frac{e}{m}}\,t \quad \text{......................(19)}$$

This is a simple harmonic motion of amplitude, y_0, and frequency, $\frac{1}{2\,\pi}\sqrt{\frac{e}{m}}$.

Next consider the case in which the supports are not fixed but execute a definite motion of their own, say, $x = x_0 \sin \omega t$ (see Fig. 7). Writing the differential equation of motion of the mass with respect to its position of rest:

$$m\,\frac{d^2y}{d\,t^2} = -\,e\,(y + x) \quad \text{......................(20)}$$

However, the author seems to be interested primarily in the relative deflection of the top and bottom of the first-story columns in a building undergoing vibration which, in his mass and spring analogy, corresponds to the deflection of the mass from the moving support, rather than to the deflection from the

position of rest. To find the relative motion of the mass with respect to the moving support it is seen from Fig. 7 that the movement relative to the support is $y + x$. Designating this motion by z: $z = y + x$, or $y = z - x$.

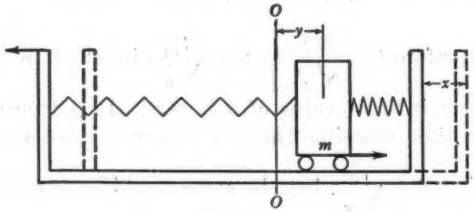

FIG. 7.

Substituting for y in Equation (20): $\frac{m\, d^2 (z - x)}{d\, t^2} = e\, z$, or,

$$m \frac{d^2 z}{d t^2} = m \frac{d^2 x}{d t^2} - e z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

which is the differential equation for the relative motion between the mass and the support. This corresponds to Equation (1) of the author's paper, except that where, in Equation (21), $\frac{d^2 x}{d\, t^2} = - x_0\, \omega^2 \sin \omega t$, he has chosen a function, $\frac{d^2 x}{d\, t^2} = a \left(1 = \frac{t}{T}\right)$, apparently representing the acceleration of an earthquake more closely than simple harmonic motion. The author's notation for y seems to be at fault. As the writer has shown, Equation (1) gives the motion of the mass relative to the support, which is what the author desires; but y in Equation (1), corresponding to z in the writer's Equation (21), is defined to be the deflection from the position of rest, instead of the relative deflection from the support.

It is to be noted that the natural frequency, or frequency of free vibration, of the system is obtained directly in solving the differential equation of motion and is, for the case considered, $F = \frac{1}{2\pi}\sqrt{\frac{e}{m}} = \frac{1}{2\pi}\sqrt{A}$, using the author's notation for A. The period of free vibration is simply, $T = \frac{1}{F} = 2\pi\sqrt{\frac{m}{e}}$ $= 2\pi \frac{1}{\sqrt{A}}$.

It is not clear what the author had in mind in differentiating Equation (8) of his paper, and in setting the first derivative equal to zero in order to get the period of free vibration. From what has been shown herein, ω, corresponding to $\sqrt{A}$ in the author's Equation (8), is determined directly in solving the differential equation; and T is derived from the fundamental relations, $\omega = 2\pi F$ and $T = \frac{1}{F} = \frac{2\pi}{\omega}$.

Differentiating Equation (8), setting the resultant equation equal to zero, and solving for t, merely gives those values of t for which the velocity of the mass is zero: $\frac{dy}{dt} = 0 = - Y_0 \sqrt{A} \sin \sqrt{A}\, t$, whence, t must be such that $\sin \sqrt{A}\, t = 0$. Therefore, $t = \frac{n \pi}{\sqrt{A}} = n \pi \sqrt{\frac{m}{e}}$, in which, $n = 0, 1, 2, 3$, etc., since the sine of any integral value of π is zero. It is true that the natural period will be the time elapsing between two successive zero points of the velocity; that is, $T = 2 \pi \sqrt{\frac{m}{e}}$, but the same would be true for zero points of the amplitude, and it is unnecessary to differentiate at all.

The author's conclusion that the maximum deflection produced in a structure by a given ground motion is not much affected by considerable changes in the period of free vibration of the structure is true only if the range of the periods of free vibration is sufficiently far above the period of the ground motion to avoid all possibilities of resonance.

Until more has been learned about the dynamics of building frames, the writer feels that the assumption that the upper stories of a building with flexible first-story columns constitute a rigid mass is a doubtful one. Simple tests made by the writer on models of various story heights show that the elasticity of the upper stories has a decided effect on the periods of free vibration, as is to be expected from purely theoretical considerations, even when the first-story columns are very flexible with respect to the upper story columns.

The writer believes that Mr. Green should be commended for presenting a method for finding the effect of any arbitrary non-harmonic vibration upon an idealized flexible building. To date, chiefly simple harmonic motions have been used in practical calculations on structures.

EDWARD J. BEDNARSKI,[4] ASSOC. M. AM. SOC. C. E. (by letter)[4a]. The author has raised an important question in the analysis of structures, namely, the influence of the connection between the structure and the foundation. He could have chosen several hypothetical variations of Fig. 2 to analyze a structure as, for example, a fixed connection resisting the forces acting in any direction and any moment created by these forces; a hinged connection that can not resist the moment and can take only the forces passing through the center of the hinge; and a connection on rollers or balls that may resist any moment, but takes only the forces directed normal to the plane of rolling.

In the case analyzed in the paper the connection is assumed of this latter type with elastic buffers preventing the structure from being carried away under occasional lateral pressure (see Fig. 2). The author could also have used the arrangement shown in Fig. 8, in which elastic springs are replaced by hydraulic "buffers" interconnected by a pipe and a dash-pot arrangement. This may be considered as an aperiodic connection if elastic expansion of the

[4] Structural Engr., Pasadena, Calif.

[4a] Received by the Secretary March 24, 1934.

pipes and the cylinders is assumed to be negligible. Of course, an absolutely elastic or absolutely aperiodic arrangement does not exist.

Connections of this kind may be likened to electric transformers in which the primary and secondary coils are connected with different degrees of loose-

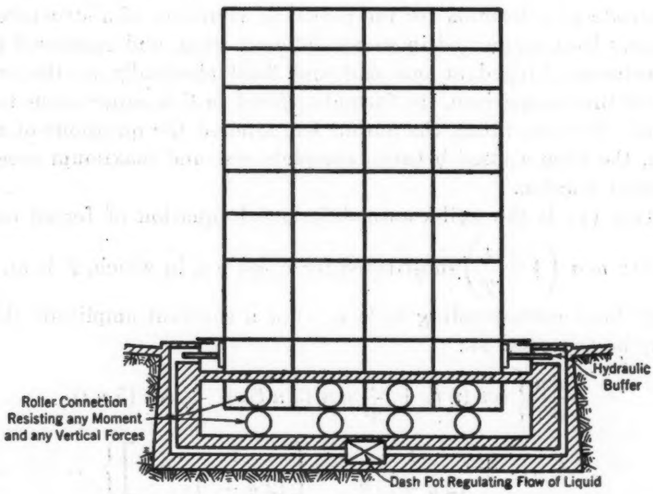

FIG. 8.

ness, in an electrical sense. In a very loose connection, this transformer will oscillate only at a certain specific frequency; that is, the natural frequency of the coupled circuit. No other frequency would create an oscillatory current.

Theoretically, a structure resting on balls or rollers, devoid of friction, does not re-act to the vibratory movement of an earthquake in a horizontal plane. In practice, however, there may be a possibility of strains due to transferred vibrations. The more flexible the first story of the structure the less will be the lateral forces imparted to it; and the less elastic its connection with the foundation, the less will be the danger of resonant vibrations being set up in it.

A connection between the structure and the foundation can not be made plastic and flexible by introducing comparatively slender columns in the first story. The use of a material with a very small elastic modulus and a relatively great plasticity between the structure and the foundation may serve the purpose in a practical sense, if the total shear resistance of the insulating material is great enough to prevent the structure from being moved by the wind pressure. The value of such shear resistance is the critical, maximum, lateral force that could ever be imparted to the structure by the earthquake without the occurrence of resonance in the horizontal plane. Common examples that illustrate this point are the old trick of snatching a napkin from beneath a number of full tumblers without spilling the contents, and the bullet passing through a pane of glass, making a clean round hole in it. The success

of the tricks depends entirely on the velocity of movement, on the friction between the napkin and the tumblers, and on the shearing resistance of glass.

In his study[5] of the theory of sound, Sir John Rayleigh introduces quite an extensive analysis to determine the natural period of vibration for a single bar, fixed at one end and free at the other. In his investigation, Mr. Green introduces a formula for the period of vibration of a structure consisting of many bars, connected in many different ways, and supported by a system of columns, hinged at one end and fixed elastically at the other. In the light of this comparison, the formula offered in this paper seems too simple to be true. For one thing, the author has ignored the questions of modes of vibration, the time elapsed between complete rest and maximum acceleration, and internal friction.

Equation (1) is the well-known differential equation of forced oscillatory motion, with $m\,a\left(1-\frac{t}{T}\right)$ substituted for $F \cos n\,t$, in which, F is an external oscillatory force corresponding to $m\,a$. For a constant amplitude this movement may be expressed as:

$$\frac{8\,m\,a}{\pi^2}\left\{\cos(n\,t)+\frac{1}{3^2}\cos(3\,n\,t)+\frac{1}{5^2}\cos(5\,n\,t)+\ldots+\frac{1}{(2\,q+1)^2}\cos\left[\frac{1}{(2\,q+1)}\,n\,t\right]\right\}\ldots\ldots\ldots\ldots(22)$$

in which, $n=\frac{2\,\pi}{4\,T}$ and q approaches ∞, $2\,q+1$ being an odd number.

The actual force imparted, at the very beginning, to the structure supported by a flexible connection will be always within the limits of 0 and $m\,a$ for absolutely flexible connection and for absolutely rigid connection. This depends on the value of e, the rate at which the value of a changes, as well as the final value of a.

From the accelerogram, Fig. 1, it seems that the amplitude of the acceleration decreases uniformly from its maximum, to an almost negligible, value, within 55 sec. This value of the amplitude may be expressed as $a\,(1-s\,t)$, in which, s = a decrement coefficient. For $t=55$ sec, $1-55\,s=0$; or $s=\frac{1}{55}$. Then the differential Equation (1) will take the form,

$$\frac{m\,d^2y}{d\,t^2}+r\,\frac{dy}{dt}+e\,y=\frac{8\,c\,m\,a}{\pi^2}(1-s\,t)\left\{\cos(n\,t)+\frac{1}{3^2}\cos(3\,n\,t)+\frac{1}{5^2}\cos(5\,n\,t)\ldots+\frac{1}{(2\,q+1)^2}\cos\left[(2\,q+1)\,n\,t\right]\right\}\ldots\ldots(23)$$

This, the writer believes, is a more general formula for the problem. The correction factor, c, which is introduced to allow for the flexibility of the connection, is not a constant, of course; but it may be considered constant betweeen certain limits. Likewise, the damping factor, r, is introduced to allow for the influence of internal frictional resistance.

[5] "The Theory of Sound," by John William Strutt Rayleigh, Second Edition, 1894–96, Vol. I, pp. 255–305.

When the connection is constructed so that the maximum force which can be imparted to the structure at the beginning is known beforehand the only danger lies in the possibility of synchronization of the vibration of the foundation with the vibration of the structure itself. It may be solved by the use of Equation (23), in which, $e = n_0^2\, m$ (in which, $n_0 = \dfrac{2\,\pi}{t_0}$ and t_0 is the time of the natural period, determined independently), and if the solution is not too complicated. In the latter case the check to determine the possibility of resonance may be made by the method introduced[6] by R. R. Martel, M. Am. Soc. C. E.

The author deserves credit for introducing a step-by-step method of solving this problem. The interval of 7 sec chosen in the numerical example ("Application of Analysis") compared with 3.58 sec of the natural period for the structure includes less than two periods and it is not quite clear why a greater maximum than $f = 0.445$ ft would not occur. Were the computations extended a little further showing that the curve of maximum values of f is descending after 7 sec, the assertion regarding the maximum deflection would carry more weight.

MERIT P. WHITE,[7] ESQ., and PAUL L. KARTZKE,[8] ESQ. (by letter)[8a].—Although investigations have been carried on for several years at various laboratories in the United States and in Japan, there is still considerable to be learned in regard to the effect of earthquakes on structures. The author's assumptions that seismic acceleration is a linear function of time and that the initial shock is sudden seem as reasonable as any others that might be made for his analysis. Actually, the final results will be practically independent of either assumption. However, the evidence of an eye-witness of one of the Japanese earthquakes that copper coins in a can were thrown out at the first shock is not proof of a sudden initial acceleration unless the can happened to be so connected to the ground as to follow the ground motion.

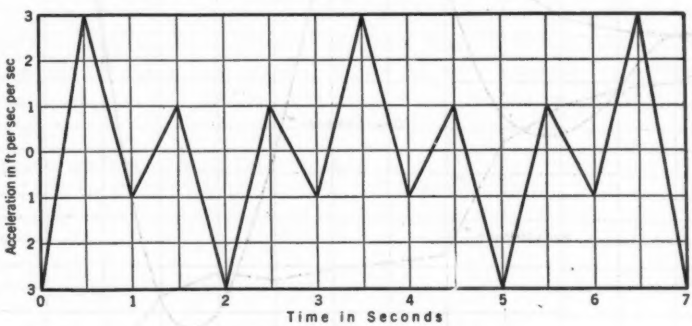

FIG. 9.—ACCELERATION DIAGRAM

[6] "The Dynamic Behavior of Some Simple Bents Subjected to Established Harmonic Motion," by R. R. Martel, M. Am. Soc. C. E., *Proceedings*, World Eng. Congress, Tokyo, Japan, 1929, Vol. VIII, pp. 213–217.

[7] Asst. in Eng., California Inst. of Technology, Pasadena, Calif.

[8] Student, California Inst. of Technology, Pasadena, Calif.

[8a] Received by the Secretary April 25, 1934.

The author's use of plus and minus signs is rather confusing. His values of Y_n and V_n at the end of each half-second period (Table 1) apparently are considered positive if in the direction of the acceleration at the beginning of that period, and are reversed in sign when they become initial conditions (Y_0, V_0) for the next period. It would seem more logical to give the plus sign to deflections and velocities to the right, and the minus sign to those to the left. Incidentally, if the second integral of the author's acceleration curve is taken it will be found that the base of the building is displaced about 20 in. assuming it to start from rest.

Fig. 9 shows an acceleration diagram somewhat similar to Fig. 3, except for the presence of a rather long period acceleration ($3\frac{1}{2}$ sec), in addition to the shorter waves. This acceleration pattern was applied to Mr. Green's building. The computations, based on the following equations,

$$Y_n = 0.639\ Y_o + 0.4385\ V_o + 0.202\ \frac{a}{T} - 0.117\ a \quad \text{.........(24)}$$

and,

$$V_n = -\ 1.350\ Y_o + 0.6392\ V_o + 0.117\ \frac{a}{T} - 0.4385\ a \quad \text{.......(25)}$$

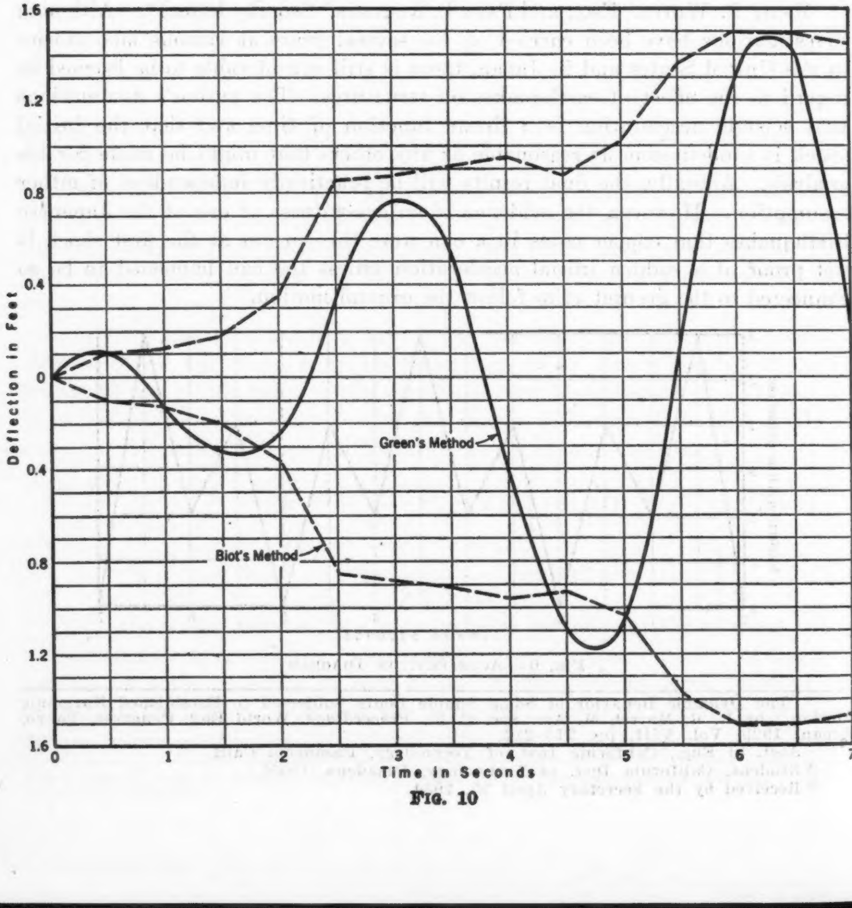

FIG. 10

yielded the deflections plotted in Fig. 10. The maximum deflection is about 1.5 ft, which is more than six times the deflection obtained by Mr. Green. Probably the next swing would be still greater.

Objection may be taken to the writers' use of a long-period acceleration, particularly in view of Mr. Green's reference to Dr. Suyehiro's conclusion that in all earthquakes the period of acceleration in a particular locality is confined to a very narrow range. An examination of the Long Beach, Vernon, and Los Angeles, Calif., records of the Long Beach earthquake of March, 1933, shows[9] the presence of waves of relatively long period. It is probable that these caused a great part of the damage to buildings with a long free period.

As an independent check on the foregoing results a calculation was made using a method of analysis[10] developed by Dr. M. Biot. This analysis was developed for a building with any number of flexible stories, in which all the stories above the first have the same flexibility. Very briefly, this method consists of considering the actual ground displacement curve, applying appropriate constants which depend on the characteristics of the building, and, by using graphical integration, obtaining an envelope (limiting curve) which will give the limits of the actual displacement curve of the floor for which calculations are made. For the flexible first-story building the equation of this envelope becomes:

$$u^2_0 = \left[\int g(t) \sin 2\,\pi\,\nu_0\,t\,dt\right]^2 + \left[\int g(t) \cos 2\,\pi\,\nu_0\,t\,dt\right]^2 \ldots. (26)$$

in which, $g(t)$ is the ground displacement curve and ν_0, the fundamental frequency of the building. The results obtained are shown in Fig. 10. Dr. Biot's method of analysis has been checked by comparison with actual deflections of a model upon a shaking-table. The agreement is quite satisfactory.

The deflections obtained in this example are obviously excessive. Furthermore, it can be seen that a careful choice of an acceleration pattern will give a similar result for any given structure. The results obtained from Mr. Green's method (or any similar method) will depend on the original choice of acceleration pattern. If one could be sure that all earthquakes in a given region would be alike, this method would be very valuable. Unfortunately, this is not the case and, hence, this method of analysis, as it stands, may be misleading.

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[9] *Engineering News-Record,* June 22, 1933.

[10] *Proceedings,* National Academy of Science, February, 1933.

APPLICATIONS FOR ADMISSION AND FOR TRANSFER

The Constitution provides that the Board of Direction shall elect or reject all applicants for *Admission* or for *Transfer,* and, in order to determine justly the eligibility of each candidate, the Board must depend largely upon the Membership for information.

This list is issued to members in every grade for the purpose of securing all such available information, and every member is urged to scan carefully each monthly list of candidates and to furnish the Board with data in regard to any applicant which may aid in determining his eligibility. It is the *Duty* of all *Members* to the *Profession* to assist the *Board* in this manner.

It is especially urged, in communications concerning applicants, that a *Definite Recommendation as to the Proper Grading in Each Case* be given, inasmuch as the grading must be based upon the opinions of those who know the applicant personally as well as upon the nature and extent of his professional experience. If facts exist derogatory to the personal character or to the professional reputation of an applicant, they should be promptly communicated to the Board. *Communications Relating to Applicants are considered by the Board as Strictly Confidential.*

The Board of Direction will not consider the applications herein contained from residents of North America until the expiration of thirty (30) days, and from non-residents of North America until the expiration of ninety (90) days from May 15, 1934.

MINIMUM REQUIREMENTS FOR ADMISSION

Grade	General Requirement	Age	Length of Active Practice	Responsible charge of work
Member	Qualified to design as well as to direct important work	35 years	12 years*	5 years of important work
Associate Member	Qualified to direct work	27 years	8 years*	1 year
Junior	Qualified for sub-professional work	20 years†	4 years*	
Affiliate	Qualified by scientific acquirements or practical experience to co-operate with engineers	35 years	12 years*	5 years of important work
Fellow	Contributor to the permanent funds of the Society			

*Graduation from a school of engineering of recognized reputation is equivalent to 4 years of active practice.

†Membership ceases at age of 33 unless transferred to higher grade.

The fact that applicants give the names of certain members as references does not necessarily mean that such members endorse.

FOR ADMISSION

BEACH, GEORGE OLIVER, Sioux City, Iowa. (Age 28.) Refers to R. A. Caughey, F. Kerekes.

BESPALOW, EUGENE FREDERICK, Memphis, Tenn. (Age 34.) Cons. Engr., Tri-State Culvert Mfg. Co., Engr. and Mgr., Hollywood Concrete Pipe Co. and Sales Engr., Choctaw Culvert & Machinery Co. (subsidiaries). Refers to E. W. Bauman, A. T. Goldbeck, H. F. Gonnerman, A. M. Lund, K. W. Markwell, G. W. Miller, F. E. Turneaure.

BLAIR, JOHN CAMBELTON, Corsicana, Tex. (Age 24.) Chf. of Party, U. S. Coast & Geodetic Survey, Civil Works, Control Div. Refers to M. L. Bowers, W. W. McClendon, J. T. L. McNew, J. J. Richey, B. F. Williams.

BOTTIGER, ANDREW JALMER, Knoxville, Tenn. (Age 40.) Engr.-in-Chg., sales and design, Sherman Concrete Pipe Co. Refers to C. E. Boesch, B. L. Crenshaw, H. H. Hale, M. W. Loving, D. D. McGuire, R. L. Morrison, E. R. Neff, G. C. White, W. H. Wilson.

BUCK, ROY McGARVEY, Bonners Ferry, Idaho. (Age 45.) Superv. Engr., U. S. Coast & Geodetic Survey. Refers to J. P. Congdon, I. C. Crawford, J. W. Howard, T. R. Newell, J. V. Otter.

CABANISS, WILLIAM FREDERICK EVE, New York City. (Age 28.) Refers to R. P. Black, F. C. Snow.

CAMPBELL, RICHARD TRENT, Tulia, Tex. (Age 23.) Instrumentman, Texas Highway Dept. Refers to O. V. Adams, W. H. Garrett, G. R. Johnston, J. H. Murdough, H. N. Roberts.

CARD, FRANCIS CHARLES, New York City. (Age 24.) Computer, U. S. Coast & Geodetic Survey. Refers to E. R. Cary, L. W. Clark, T. R. Lawson, P. C. Ricketts.

CHAMBERLAIN, ANTONIO ARRILLAGA, Catskill, N. Y. (Age 29.) Jun. Asst. Engr., Grade 2, Div. of Eng., New York State Dept of Public Works, Albany, N. Y. Refers to R. W. Briggs, G. E. F. Lund, J. D. Morales, H. O. Schermerhorn, L. E. Stephenson.

CHETTLE, EARL VINCENT, Brigham City, Utah. (Age 28.) City Engr. Refers to R. B. Ketchum, F. H. Richardson.

COHN, MORRIS MANDEL, Schenectady, N. Y. (Age 35.) San. and Testing Engr., Dept. of Public Works. Refers to E. S. Chase, E. Devendorf, H. P. Eddy, G. B. Gascoigne, G. D. Holmes, F. R. Lanagan, J. F. Skinner, W. C. Taylor.

COMM, EDWARD DANIEL, Anoka, Minn. (Age 22.) Ballistician, Federal Cartridge Corporation. Refers to E. F. Chandler, W. E. Smith.

COOMBE, JOHN VAN VEGHTEN, Chicago, Ill. (Age 25.) Advisor, Champaign-Urbana Belt-Line Traffic Survey. Refers to J. S. Crandell, J. J. Doland, W. C. Huntington, C. C. Wiley.

COSGROVE, ALAN GEORGE, Troy, N. Y. (Age 28.) Asst. Engr., C. W. A., U. S. Geological Survey, Water Resources Branch. Refers to A. D. Blanchard, J. Greer, A. W. Harrington, H. Johnson, J. J. Vail.

FUGATE, DOUGLAS BROWN, Radford, Va. (Age 27.) Instrumentman and Inspector, State Highway Dept., Richmond, Va. Refers to J. A. Anderson, A. H. Bell, H. G. Shirley.

GERRICK, EDWARD EARNEST, Seattle, Wash. (Age 37.) Secy.-Treas., Diatom Products Co. Refers to D. H. Evans, C. J. Hogue, W. A. Kunigk, D. W. McMorris, F. Mears, E. L. Strandberg.

GOLDSTEIN, JOSEPH, New York City. (Age 37.) Inspector on pier construction, Dept. of Docks, New York City. Refers to F. A. Ciampa, J. Feld, G. D. Fish, M. J. Gruenbaum, B. J. Mansfield, G. F. Reeves, E. E. Seelye, A. L. Stevenson, H. S. Woodward.

HANSELMAN, RICHARD MALCOLM, Rossford, Ohio. (Age 22.) Rodman and Instrumentman, George Champe and Associates. Refers to G. Champe, A. J. Decker, C. S. Finkbeiner, W. C. Hoad, H. P. Jones, W. C. Sadler.

HARVEY, LINVAL DALLAS, Swansea, Mass. (Age 21.) Refers to J. W. Howe, C. F. Meyer.

HIZA, MARTIN WILLIAM, Jr., Newark, N. J. (Age 26.) Recorder and Rodman, U. S. Coast & Geodetic Survey in New Jersey, under C. W. A. and E. R. A. Refers to J. L. Davis, R. C. Johnson, W. E. Rowe, F. W. Tooker, W. M. Van Wagner.

JOHNSTON, BRUCE GILBERT, Bethlehem, Pa. (Age 28.) Graduate Student and Research Fellow, Lehigh Univ. Refers to H. Cross, W. C. Huntington, I. M. Lyse, H. C. Neuffer, F. E. Richart, T. C. Shedd, C. H. Sutherland.

KELLY, HENRY JERVEY, Knoxville, Tenn. (Age 28.) Asst. Civ. Engr., Eng. Service Div., Tennessee Valley Authority. Refers to A. W. Parker, N. A. Saigh, C. H. Schwartz, F. C. Snow, G. D. Whitmore, H. A. Wiersema.

KENDALL, NATHANIEL JAMES, Los Altos, Cal. (Age 25.) Refers to A. C. Beyer, C. Moser, A. S. Niles, L. B. Reynolds, J. B. Wells.

LANGAN, JOHN ALBERT, Syracuse, N. Y. (Age 35.) Dist. Field Engr., New York State Temporary Emergency Relief Administration (TERA). Refers to H. B. Brewster, S. N. Grimm, G. D. Holmes, W. F. Kavanaugh, E. F. O'Brien, M. B. Palmer, N. F. Pitts, Jr.

LEONARD, JOHN JOSEPH, San Francisco, Cal. (Age 30.) Member of firm, J. J. Leonard & Son, Bldg. Contrs. Refers to A. V. Bowhay, M. J. Callaghan, E. C. Flynn, J. B. Leonard, R. H. Owens, G. L. Sullivan.

LEWIS, WILLIAM WHITFIELD, Durham, N. C. (Age 25.) Instructor in Civ. Eng., Duke Univ. Refers to H. C. Bird, P. A. Franklin, W. H. Hall, H. O. Hill, J. D. Waldrop.

MACABEE, LLOYD CEDRIC, Menlo Park, Cal. (Age 31.) Deputy County Engr., San Mateo County. Refers to A. C. Beyer, J. F. Byxbee, C. G. Gillespie, J. S. James, L. B. Reynolds, E. C. Thomas.

MARKIEWICZ, VICTOR ADAM XENON, New York City. (Age 22.) Topographical Draftsman, Dept. of Parks, City of New York. Refers to A. Haring, E. G. Hooper, C. T. Schwarze, D. S. Trowbridge.

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NEIMAN, HERMAN BURBANK, Philadelphia, Pa. (Age 36.) Field Engr. and Supt., United Engrs. & Constrs., Inc. Refers to C. D. Babcock, J. T. Kiernan, A. Miedwig, J. W. Moffett, J. L. Orr, L. A. Whitsit.

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PORRATA DORIA, FRANK LUIS, Ponce, Puerto Rico. (Age 44.) Cons. Engr. for City of Ponce. Refers to M. Font, J. M. Giles, A. S. Lucchetti-Otero, R. Ramirez, A. Rodriguez, E. Totti y Torres.

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RUSSELL, TOM, Sioux City, Iowa. (Age 34.) Asst. City Engr Refers to H. B. Christianson, P. D. Cook, R. M. Coomer, E. L. Ferguson, J. L. Holdefer.

SMITH, GILES ALBERT, Cincinnati, Ohio. (Age 28.) Asst. Supt., The Holmes Constr. Co. Refers to R. A. Anderegg, I. N. Clover, H. H. Kranz, R. M. Miller, C. Wuest, Jr.

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SOLER-LOPEZ, ERNESTO ANTONIO, Mayaguez, Puerto Rico. (Age 24.) Inspector, War Dept., U. S. Engr. Office, San Juan, Puerto Rico. Refers to W. H. Crago, M. Font, J. D. Morales, E. Mowlds, R. Ramirez, C. del Valle Zeno.

STOEFFEL, WILLIAM HENRY, Jr., New York City. (Age 28.) Asst. Engr., Bronx Park Dept., under C. W. A. Refers to C. F. Dykeman, T. F. McQuade, L. E. Robbe, J. C. Scott, J. M. C. van Hulsteyn.

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WATKINS, ERNEST MONROE, Greenville, S. C. (Age 23.) With J. E. Sirrine & Co. Refers to H. L. Hagerman, A. C. Lee, F. J. Lewis, W. S. Lindsay, J. E. Sirrine.

WEBB, WINCHELL CLIFFORD, State College, Miss. (Age 30.) Asst. Prof. of Civ. Eng., Mississippi State Coll. Refers to D. M. McCain, R. F. Rudolph, W. J. Shackelford, R. L. Totten, C. Woodfin.

WESTCOTT, CLIFFORD HARPER, Chicago, Ill. (Age 41.) Chf. Engr., Westcott Eng. Co., Structural Engrs. Refers to P. L. Hein, A. R. Lord, E. S. Nethercut, C. L. Post, T. F. Quilty, F. A. Randall, D. B. Rush.

WILSON, HARVEY ASHTON, Abbeville, La. (Age 30.) Asst. Supt., Jefferson Lake Oil Co., Inc., Sulpher Miners, Frasch Process, Barba, La. Refers to C. G. Cappel, D. Derickson, E. E. Elam, F. C. Fox, Jr., W. B. Gregory.

YOUNG, CHARLES AUGUSTUS, Ft. Smith, Ark. (Age 28.) With U. S. Geological Survey, Water Resources Branch. Refers to E. C. H. Bantel, G. H. Canfield, C. E. Ellsworth, J. H. Gardiner, J. A. Norris, T. U. Taylor, G. G. Wickline.

YUDA, WILLIAM ALEXANDER, Pearl River, N. Y. (Age 26.) Computer, U. S. Coast and Goedetic Survey. Refers to F. A. Boyle, J. J. Costa, E. P. Leclercq, G. J. Meise.

FOR TRANSFER

FROM THE GRADE OF ASSOCIATE MEMBER

DOHERTY, WILLIAM EDWARD ALOYSIUS, Assoc. M., Philadelphia, Pa. (Elected June 4, 1928.) (Age 40.) Engr. of Constr., Bureau of Highways, City of Philadelphia, Pa. Refers to J. Adler, J. H. M. Andrews, C. E. Myers, J. H. Neeson, C. S. Shaughnessy.

FELD, JACOB, Assoc. M., New York City. (Elected Junior June 19, 1922; Assoc. M., May 19, 1924.) (Age 35.) Cons. Engr. Refers to S. Bernstein, H. M. Braloff, H. Goldmark, G. Paaswell, D. B. Steinman, A. W. Stephens.

FULLER, RAYMOND STILES, Assoc. M., Berkeley, Cal. (Elected Dec. 4, 1922.) (Age 45.) Engr. of Gas Distribution, Pacific Gas & Elec. Co., San Francisco, Cal. Refers to B. C. Gerwick, L. M. Klauber, A. H. Markwart, F. R. Muhs, J. T. Ryan, C. W. Schedler, Jr., I. C. Steele.

GREGOR, MICHAEL, Assoc. M., New York City. (Elected March 5, 1928.) (Age 46.) Cons. Engr. and Executive Engr., Seversky Aircraft Corporation. Refers to F. W. Altstaetter, S. P. Maximoff, C. P. Melioransky, L. S. Moisseiff, C. S. Nichols, V. E. Timonoff, R. H. Wilson.

HARRELL, CHARLES ADAIR, Assoc. M., Binghamton, N. Y. (Elected Aug. 17, 1931.) (Age 40.) City Mgr. Refers to E. D. Gilman, H. H. Kranz, H. B. Luther, H. Schneider, C. O. Sherrill.

HEIM, ARTHUR IRVING, Assoc. M., New York City. (Elected Junior Jan. 17, 1921; Assoc. M. Feb. 25, 1924.) (Age 38.) Chf. Draftsman with A. E. Wheeler, Cons. Metallurgical Engr. Refers to B. J. Ahearn, R. B. Brushaber, E. G. Haines, A. H. Jorgensen, S. O. Rogde, W. M. Spann, J. D. Ward.

LAVERTY, FRANCIS JOHN, Assoc. M., Pleasantville, N. Y. (Elected Junior Oct. 12, 1925; Assoc. M. April 23, 1928.) Age 35.) Project Engr., Westchester County San. Sewer Comm.; Lieut, Engr. O. R. C. Refers to E. A. Andrews, A. M. Brosius, J. M. Duffy, E. H. Feldmann, J. F. Halpin, C. A. Latimer, J. Muss, W. J. Shea, C. J. Sheridan.

LEWIS, ISIDORE LEONARD, Assoc. M., Philadelphia, Pa. (Elected Jan. 17, 1927.) (Age 47.) Senior Draftsman, Dept. of City Transit. Refers to H. S. Hipwell, S. S. A. Keast, W. Linker, S. Napp, C. W. Palmer, S. I. Sacks, C. H. Stevens, L. Zislin.

MIRABELLI, EUGENE, Assoc. M., Arlington, Mass. (Elected April 7, 1924.) Age 35.) Asst. Prof. of Structural Design, Massachusetts Inst. of Technology, Cambridge, Mass. Refers to H. K. Barrows, H. L. Bowman, C. A. Farwell, G. E. Russell, C. M. Spofford, C. H. Sutherland, R. G. Tyler.

SANTELMANN, ALFRED WILLIAM, Assoc. M., Washington, D. C. (Elected Junior May 28, 1923; Assoc. M. Nov. 14, 1927.) (Age 36.) Associate Structural Engr., Superv. Archt.'s Office, Treasury Dept. Refers to W. S. Anderson, J. W. Dunham, D. Ferguson, R. G. Focht, N. H. Leavitt.

TOZZER, CARL HENRY, Assoc. M., Marion, Ohio. (Elected March 15, 1926.) (Age 44.) County Surveyor, Marion County, Ohio. Refers to F. G. Browne, A. A. Burger, G. B. Gascoigne, W. L. Havens, C. E. Swank.

FROM THE GRADE OF JUNIOR

BENES, EDWARD WILLIAM, Jun., Mt. Vernon, N. Y. (Elected Dec. 3, 1926.) (Age 32.) Asst. Engr. with A. Burton Cohen, Cons. Engr., New York City. Refers to A. B. Cohen, F. W. Dencer, E. L. Eriksen, W. K. Hatt, N. A. Richards, E. M. Vincent.

BOOTH, DONALD PRENTICE, Jun., Rock Island, Ill. (Elected March 11, 1929.) (Age 31.) Lieut., U. S. Army; Asst. to Dist. Engr., Operations Div. Refers to R. E. Coughlin, E L. Daley, L. C. Urquhart, E. C. Webster, H. J. Wild.

BUCHMUELLER, MILTON, Jun., St. Louis, Mo., (Elected Feb. 25, 1924.) (Age 32.) Civ. Engr., Supply and Purifying Sec., St. Louis Water Div. Refers to C. S. Bumann, C. M. Daily, J. B. Dean, E. E. Easterday, H. R. J. Meyer, J. C. Pritchard, E. E. Wall.

BUCK, HENRY WOLCOTT, Jun., Hartford, Conn. (Elected Dec. 14, 1925.) (Age 31.) Member of firm, Henry Robinson Buck, Inc. Refers to C. J. Bennett, T. Crane, R. J. Ross, W. J. Scott, R. H. Suttie, W. A. D. Wurts.

EVERS, WILLIAM HENRY, Jr., Jun., Cleveland, Ohio. (Elected Oct. 12, 1925.) (Age 31.) Cons. Engr., Wm. H. Evers and Associates. Refers to W. H. Evers, J. M. Heffelfinger, Jr., R. Hoffmann, F. A. Pease, W. E. Pease, C. W. Root, G. B. Sowers, A. W. Zesiger.

LAWTON, MILTON BURBANK, Jun., Scarsdale, N. Y. (Elected July 11, 1927.) (Age 32.) Jun. Engr., New York State Transit Comm., New York City. Refers to W. S. L. Cleverdon, C. W. Coote, N. P. Gerhard, H. M. Leon, W. L. Selmer, J. F. Williamson.

MIMS, STALEY WOOD, Jun., Bronson, Tex. (Elected Aug. 15, 1932.) (Age 30.) Res. Engr., Texas State Highway Dept. Refers to D. K. Caldwell, T. E. Huffman, V. G. Koch, J. T. L. McNew, E. J. Nichols.

OOTHOUT, RAYMOND MILTON, Jun., Des Moines, Iowa. (Elected Dec. 3, 1928.) (Age 27.) Constr. Engr., Koss Constr. Co. Refers to E. L. Erickson, I. G. Hedrick, G. W. Koss, V. H. Smith, J. M. Tippee.

PEARSON, THEO PORTER, Jun., Conway, S. C. (Elected Aug. 15, 1932.) (Age 30.) Contract Estimating Engr. and Asst. Mgr., Standard Dredging Co., New York City. Refers to W. S. Fitzsimons, W. F. Lineberger, G. W. Sackett, M. J. Young, G. A. Youngberg.

PERLITER, SIMON, Jun., Los Angeles, Cal. (Elected Oct. 12, 1925.) (Age 32.) Asst. Engr., Metropolitan Water Dist. of Southern California. Refers to A. R. Baker, C. A. Bissell, J. B. Bond, W. C. Christopher, J. Hinds, J. Munn, F. E. Weymouth.

SCHIRMER, HOWARD AUGUST, Jun., Oakland, Cal. (Elected May 19, 1924.) (Age 32.) Structural Engr., McClintic-Marshall Corporation. Refers to M. C. Collins, C. Derleth, Jr., E. L. Durkee, A. W. Earl, P. A. Franklin, D. S. Gendell, Jr., B. Jameyson, J. Jones.

SENSING, WILBUR CARSON, Jun., Nashville, Tenn. (Elected Nov. 15, 1926.) (Age 32.) Engr. and Secy., Wilson-Weesner-Wilkinson Co. Refers to E. W. Bauman, R. N. Coolidge, H. B. Dyer, C. N. Harrub, F. J. Lewis, E. B. Wilkinson, C. B. Wilson.

VAN REEKUM, VERNON JOHN, Jun., Dallas, Tex. (Elected March 10, 1930.) (Age 32.) Civ. Engr., City of University Park. Refers to E. L. Myers, E. N. Noyes, F. W. Pearce, I. C. Peterson, A. R. Webb.

The Board of Direction will consider the applications in this list not less than thirty days after the date of issue.